AN APPLICATION OF SEASONAL COINTEGRATION AND ERROR CORRECTION MODELS ON MONTHLY DATA

A Thesis
Submitted to the Department of Economics
and the Institute of Economics and Social Sciences of
Bilkent University
In Partial Fulfillment of the Requirements
for the Degree of

MASTER IN ECONOMICS

by
Güliz ERCOŞKUN
July 1995

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tarafından bağışlanmıştır.

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I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master in Economics.

Asist. Prof. Dr. Kıvılcım Metin

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master in Economics.

Prof. Dr. Asad Zaman

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master in Economics.

Asist. Prof. Dr. Osman Zaim

Approved by the Institute of Social and Economic Sciences

Director:

ABSTRACT

AN APPLICATION OF SEASONAL COINTEGRATION AND ERROR CORRECTION MODELS ON MONTHLY DATA

Güliz Ercoşkun
M in Economics
Supervisor: Asist. Prof. Dr. Kıvılcım Metin
June 1995

In this study, I try to analyze and show the monthly changes and their effects on each other of Istanbul Stock Exchange (ISE), TL / $ Exchange Rate (E), M1, M2, price level (P), Interest rate on securities (R) and Advances of the central bank to the treasury (A) by developed techniques in time series econometrics, namely unit roots, seasonal cointegration and error correction. The long run relationship between stock prices and exchange rate, price level, M1, M2 investigated by using these techniques of time series. Conclusions are made for future use of models for monthly time series. To our knowledge, this is among the pioneering studies conducted in an emerging market that uses an updated econometric methodology to allow for an analysis of monthly data for long run steady state properties together with short run dynamics.

Key Words: Unit Root, Seasonal Cointegration, Error Correction, Istanbul Stock Exchange.

ÖZET

MEVSİMSEL KOİNTEGRASYON VE HATA DÜZELTME MODELLERİNİN AYLIK VERİLER ÜZERİNE UYGULANMASI

Güliz Ercoşkun
Yüksek Lisans Tezi
Tez Yöneticisi: Yrd. Doç. Dr. Kıvılcım Metin
Temmuz 1995

Bu tez Türkiye'de 1986-1994 dönemindeki İstanbul Menkul Kıymetler Borsası, döviz kuru (TL\$), M1, M2, enfilasyon, avans ve hazine bonosu faiz oranları arasındaki ilişkileri aylık veriler göz önüne alınarak incelemektedir. Ekonometrik olarak, zaman serileri kullanılarak aylık veriler için mevsimsel kointegrasyon ve hata düzeltme modelleri türetilmiştir. Bu calışma, bu konuda aylık veriler baz alınarak hazırlanmış öncü çalışmalardan birisidir.

Anahtar Kelimeler: Birim kök, Mevsimsel Kointegrasyon, Hata Düzeltme, İstanbul Menkul Kıymetler Borsası

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CONTENTS

CHAPTER 1

1.1 INTRODUCTION

An important set of information which is ignored in the efficient market literature - with only a few exceptions - is the information revealed by macroeconomic variables. Fama (1991) concludes his well known review article on efficient markets by encouraging research that "relates the behavior of expected returns to the real economy". Macroeconomic variables constitute a relatively more important set of information in thin markets in comparison to mature ones. In thin markets, the volume of trade is relatively low, and publicity available information on company performances is generally limited and untimely. Also, most of the thin markets are operational in developing countries where capital accumulation and economic activity is initiated by the state. Therefore, the thinly traded stock markets of controlled economies are expected to absorb fiscal and monetary changes as important sets of information.

The conventional methodology employed in this field of research, briefly reviewed above, is based on the use of time series regression. The development of seasonal cointegration theory in econometrics permits a long-run analysis of the nonstationary time series to study the relationship between stock returns and macroeconomic variables, using an error correction model of stock prices and testing for the seasonal cointegrating relation between stock prices and the variables of interests.

In this study, I try to test the relationships between Istanbul Stock Exchange (ISE) and price level (P), M1, M2, Interest Rates (R), Advances of central bank to treasury (A), Turkish Lira-dollar exchange rate (E) by using the time series analysis namely, seasonal cointegration and error correction. To our knowledge, this is among the pioneering studies conducted in an emerging market that uses an updated econometric methodology to allow for an analysis of long run steady state properties together with short run dynamics for monthly data.

Accordingly, the thesis is organized as follows. After presenting a brief description of unit root, seasonal cointegration, I deal with seasonal cointegration and error correction theory for monthly data for Istanbul Stock Exchange and other variables. First of all, the variables which are used are driven, then the Hylleberg-Engle-Granger-Yoo (HEGY) model for quarterly data improved and made useful for monthly data. Then these tests are used to test the cointegration relationships among the variables. The existence of long run equilibrium relations were tested by updated version of Engle and Granger (1987) two-step approach, Frances (1991) seasonality approach and HEGY (1990) seasonal cointegration and error correction approach, but here I want to point out again that these models are updated for monthly case. Evidence is provided for long run movements of macro-economic variables and stock prices as well as their short run behavior. Finally, conclusions are made and a very useful method for testing of seasonal cointegration and error-correction models for time series are derived for monthly case.

1.2 THE SETTINGS

Turkey for more than a decade has functioned as a good case study for the set of developing and post-communist countries in the process of structural change and liberalization. Structural change from a government - regulated economic regime to a market-oriented one commenced with the economic package introduced in January 1980. Main topics of the policies were the convertibility of the Turkish lira, flexible exchange rate policy and export promotions. As a component of the program, there was a major devaluation of Turkish lira in January 1980. The 1980 program also included some interest rate policy, which lead interest rates exceed inflation rate. As a result of these policies in 1981-1983 period, the inflation rate did not exceed 36%.

In the period 1984-1987, the average inflation rate was around 40%. In April 1986, the Central Bank set up an Interbank market for one and two week maturities and introduced overnight transaction in May 1986. In 1986, the Central Bank introduced for the first time the policy approach of targeting a monetary aggregate. Money in wider sense (M2) was selected to be kept on a growth path during the year. In 1986, M2 grew 38.6%, which was close to the target level. In 1986, M1 had a growth of 62.5% and reserve money had a growth of 32.8% and the consumer price inflation achieved 34.6%. For 1987 the monetary authorities targeted growth of M2 at 30 percent which was considered consistent with an expansion of 5 percent and an inflation rate of 25 percent. The central Bank planned 28 percent growth of the reserve money which was the main instrument to control M2. But reserve money growth was nearly 50 percent in 1987 and consumer price inflation was 38.9 percent. In 1987, M1 growth was 58.3% and M2 growth was 37.6%.

In view of accelerating inflation and instability in financial markets, monetary policy was severely tightened in 1988. Deposit interest rates were raised to encourage financial savings and to reduce the share of currency and sight deposits in M2. But, besides this tightening policy, targets were exceeded by substantial amount in 1988. M1, M2 and reserve money growth were 39.7%, 77.5% and 67.5%, respectively. Consumer price inflation reached 75.4% in 1988.

For 1989, the Central Bank has abstained from announcing monetary targets. In 1989, reserve growth accelerated due to increase in net foreign assets and due to the government's decision to grant large salary increases and to raise agricultural support prices. Reserve money growth reached 75% and M1 and M2 growth were 97.1% and 82%, respectively. In 1989, consumer price inflation was at the level of 69.9%. In the context of the program of economic liberalization, the Turkish authorities have been aiming at placing greater reliance on monetary policy for economic stabilization purposes. However, as the Central Bank is not completely autonomous and economic policy decisions are taken at the governmental level, it has been difficult to follow a clear anti-inflationary monetary policy.

Starting from 1990, interest rate-exchange rate balance and foreign capital inflow have directly depended on each other. In 1990 return from interest was 2.5% above the return from foreign currency and this caused 3000 million dollars of foreign capital inflow. In 1991, the return from interest over return from foreign currency fell to -3.3% and this caused 3020 million dollars of capital to leave the country. From this time after, return from interest have been always above the return from foreign currency and in 1992 and 1993 there have been seen net foreign capital inflow. In 1993, total Capital Movements item has reached 9279 million dollars and this value is 5.6% of GNP in 1993.

Inflation has reached an average of 68.2% in the period 1988-1992. Monetary policy aimed at maintaining orderly conditions in financial markets. The Central Bank, however, was again obliged to finance the PSBR, and hence fiscal imbalance induced rapid growth in monetary aggregates. In the period 1988-1992, M1, M2 and reserve money growth reached an average of 62%, 67% and 58%, respectively.

Strong output growth in 1992 and 1993, led by domestic demand, brought about a widening current account deficit and rising foreign indebtedness. Inflationary pressures intensified, partly in response to further increase in public sector deficits to very high levels. In 1993, real GNP growth averaged 6.75%, the trade deficit rose to 12% of GNP and public sector borrowing requirement (PSBR) rose to 16% of GNP. Annual consumer price inflation averaged 66% in 1993, compared with 70% in 1992. At the end of 1993, international credit worthiness was downrated and the Turkish lira drastically depreciated. M1, M2 and reserve money growth were 53%, 43% and 60%, respectively in 1993.

Starting in 1994, Turkish economy have undergone the most important crisis of the last 15 years. The crises has started in the first months of 1994 in finance market and it has spread to real part of the economy in a little time. The main causes of the crises has been shown as the growing public sector deficits and the incorrect steps towards liberalization.

For this period, consumer price inflation was 126%, and wholesale price inflation was 150% in 1994. Public sector borrowing requirement fell to 8% of GNP. In 1994, M1, M2 and reserve money growth reached 85%, 132% and 85%, respectively. In April 1995, the annual consumer price inflation achieved 94%. And in April 1995, the three months M1, M2 and reserve money growth ratios achieved 15.6%, 19.2% and 20%, respectively.

1.3 THE DATA SET

Our data set consists of monthly observations for the period 1986:1-1994:12; all the observations are as the end of period. Considering the macroeconomics of the Turkish economy, we have set the relations between stock returns and a set of macroeconomic variables and I choose my variables according to this.

Stock returns are represented by the monthly index value of the Istanbul Stock Exchange (ISE). Considering the relationship between inflation and the budget deficit (Metin, 1993,1994) this variable is included in the data set. Budget deficit is represented by the advances of the central bank to the treasury (A) because the budget deficit is not announced on a monthly basis and these advances are widely used in the financial media as indicators of the annual budget deficit. Other variables are also chosen on the basis of the availability and the higher frequency of use of information by the ultimate investors. Interest rates (R) are depicted by the monthly compounded value of the three month treasury bill rate which is sensitive measure of the "going rate of interest" in the financial media. The Turkish lira-U.S. dollar exchange rate (E) is also included in the data set due to the frequent open market operations of the Central Bank using dollar reserves. Inflation (P) is measured by the consumer price index. Finally, money supply is represented by two monetary aggregates; M1 which is currency in circulation plus demand deposits and, M2 which is M1 plus time deposits. All data are collected from several issues of the Three Monthly Bulletin of the Turkish Treasury. None of the series are seasonally adjusted.

CHAPTER 2

2.1 DEFINITION

The rapidly developing time-series analysis of models with unit roots has had a major impact on econometric practice and on our understanding of the response of econometric systems to shocks.

Many economic time series contain important seasonal components and there are a variety of possible models for seasonality which may differ across series. A seasonal series can be described as one with a spectrum having distinct peaks at the seasonal frequencies:

$w_s = 2\pi jk / n$ where $k = 0, 1, \ldots, n-1$, w_s are seasonal frequencies and n is the number of periods in a year.

Three classes of time-series models are commonly used to model seasonality. These can be called;

(a) Purely deterministic seasonal process,

(b) Stationary seasonal process,

(c) Integrated seasonal process.

A purely deterministic seasonal process x_t is a process generated by seasonal dummy variable such as;

$x_t = \mu_t$, $\mu_t = m_0 + m_1S_1 + m_2S_2 + ... + m_{k-1}S_{k-1}$ k = number of periods per year,

m_i = constants, S_i = seasonals.

this process can be perfectly forecasted and will never change its shape.

A stationary seasonal process $\Psi(B)$ can be generated by a potentially infinite autoregression.

$\Psi(B)x_t = \varepsilon_t$, $\varepsilon_t \sim$ i.i.d, independently identically distributed

with all of the roots of $\Psi(B) = 0$ lying outside the unit circle but where some complex pairs with seasonal periodicities. More precisely, the spectrum of such a process is given by:

$f(w) = \sigma^2 / | \Psi(e^{jw}) |^2$ where σ^2 is some constant.

A series x_t is an integrated seasonal process if it has a seasonal unit root in its autoregressive representation. More generally, it is integrated of order d at frequency θ if the spectrum of x_t, takes the form:

$$f(w) = c(w - \theta)^{-2d}$$

for w near θ. This is conveniently denoted by:

$$x_t \sim I_\theta (d).$$

So, a series with a clear seasonal may be seasonally integrated, have a deterministic seasonal, a stationary seasonal, or some combination. A general class of linear time-series models which exhibit potentially complex forms of seasonality can be written as:

$$d(B)\, a(B)\, (x_t - \mu_t) = \varepsilon_t$$

where all the roots of $a(z) = 0$ lie outside the unit circle, all the roots of $d(z) = 0$ lie on the unit circle, and μ_t is some constant. Stationary seasonality and other stationary components of x_t are absorbed into a(B), while deterministic seasonality is in μ_t when there are no seasonal unit roots in d(B).

2.2 IMPORTANCE OF RIGHT AUGMENTATION

Suppose that each component of x_t is I(1) so that the change in each component is a zero mean purely nondeterministic stationary stochastic process. Any known deterministic components can be subtracted before the analysis is begun. It follows that there will always exist a multivariate Wold representation such that:

$$(1\text{-}B)\, x_t = C(B)\, \varepsilon_t,$$

If we take to mean of both sides, we will have the same spectral matrix. Further, C(B) will be uniquely defined by the conditions that the function det[C(z)], $z = e^{jw}$, have all zeroes on or outside the unit circle, and that $C(0) = I_N$, the N X N identity matrix. In this representation the ε_t has zero mean white noise vectors with:

$$E[\varepsilon_t \varepsilon_t'] = 0, \quad t \neq n,$$
$$= G, \quad t = n,$$

so that only contemporaneous correlations can occur.

Due to habit and convenience ε_t is often assumed to be i.i.d. or n.i.d. It is quite easy to show that a process such as $(1\text{-}B^{12})y_t = \varepsilon_t$ or $(1\text{+}B)y_t = \varepsilon_t$ has property that a process may alter the seasonal pattern completely. The definition of integration does not require ε_t to be anything else than stationary, in fact ε_t can be bounded, heteroskedastic, autocorrelated conditional heteroskedastic, autocorrelated, nonsymmetric, etc. Due to the complex nature of the economic system, i.e., of the data-generating process ε_t , a simple univariate representation such as $(1\text{-}B^{12})y_t = \varepsilon_t$ may be expected to display such behavior. Such a univariate representation is therefore also best seen as an approximation that should be interpreted with care. Here an integrated seasonal model is applied to data with a varying seasonal pattern and the choice between a deterministic seasonal model and integrated model depends on the degree of variation the seasonal pattern.

The auxiliary regression may be augmented by lagged values of the dependent variable $(1-B^{12})y_t = \varepsilon_t$ without an effect on the distribution under the null as is the case with the Dickey - Fuller procedure. However, the power and size of the test may depend critically on the 'right' augmentation being used. From Monte Carlo experiments we know that the power of Dickey-Fuller test suffers if too many auxiliary parameters are applied to render the errors white noise, while the size may be far greater than the chosen level of significance if we use too few parameters. In addition one may add deterministic terms like an intercept, seasonal dummies, and a trend, but this will change the distribution.

Notice that our discussion has been confined to the case of i.i.d. error terms. When the error terms are intertemporally dependent, however, the limiting distributions depend on nuisance parameters, i.e., the variance of ε_{ct} at the zero frequency in our case. Again the power and the size of the test may be expected to depend critically on the right augmentation being used.

2.3 TESTING PROCEDURE

The goal of the testing procedure proposed in this theses is to determine whether or not there is any seasonal unit roots, if exists, their cointegrations and error corrections with the other variables. The test must take seriously the possibility that seasonality of every forms may be present, and at the same time, the tests for conventional unit roots will be examined in seasonal settings.

In the literature there exist a few attempts to develop such tests. Dickey, Hasza, and Fuller (1984), following the lead suggested by Dickey and Fuller for the zero-frequency unit root case, propose a test of the hypothesis $a = 1$ against the alternative $a < 1$ in the model $x_t = ax_{t-1} + \varepsilon_t$. The asymptotic distribution of the least - squares estimator is found and the small-sample distribution obtained for several values by Monte Carlo methods. In addition the test is extended to the case of higher-order stationary dynamics. A major drawback of this test is that it doesn't allow for unit roots at some but not all of the seasonal frequencies and that the alternative has a very particular form, namely that all the roots have the same modulus. In this thesis, I propose a test and a general framework for a test strategy that examines at unit roots at all the seasonal frequencies as well as the zero frequency for monthly data. The test follows the HEGY and Eagle & Granger framework and in fact has a well-known distribution possibly performed on transformed variables in some special cases.

As presented above, the goal of this testing procedure proposed in this thesis to determine whether or not there are any seasonal unit roots in time series. The test must consider the possibility that seasonality of other forms may be present. At the same time, the test for conventional unit roots will be examined in seasonal settings.

To test the hypothesis that the roots of $\varphi(B)$ lie on the unit circle against the alternative that they lie outside the unit circle, it is convenient to rewrite the autoregressive polynomial according to the following proposition which is originally due to Lagrange and is used in approximation theory.

Proposition: Any (possibly infinite or rational) polynomial $\varphi(B)$, which is finite-valued at the distinct, nonzero, possibly complex points $\theta_1, \ldots, \theta_p$ can be expressed in terms of elementary polynomials and a remainder as follows:

$$\varphi(B) = \sum_{k=1}^{p} \lambda_k \, \Delta(B) / \delta_k(B) + \Delta(B) \, \varphi^{**}(B), \tag{1}$$

where the λ_k are a set of constants, $\varphi^{**}(B)$ is a (possibly infinite or rational) polynomial, and:

$$\delta_k(B) = 1 - (1/\theta_k)B, \tag{2}$$

$$\Delta(B) = \prod_{k=1}^{p} \delta_k(B). \tag{3}$$

Proof: Let λ_k be defined to be:

$$\lambda_k = \varphi(\theta_k) / \prod_{j \neq k} \delta_j(\theta_k), \tag{4}$$

which always exists since all the roots of the δ's are distinct and the polynomial is bounded at each value by assumption. The polynomial:

$$\varphi(B) - \sum_{k=1}^{p} \lambda_k \Delta(B) / \delta_k(B) = \varphi(B) - \sum_{k=1}^{p} \varphi(\theta_k) \prod_{j \neq k} \delta_j(B) / \delta_j(\theta_k) \tag{5}$$

will have zeroes at each point $B = \theta_k$. Thus, it can be written as the product of a polynomial, say $\varphi^{**}(B)$, and $\Delta(B)$. QED

An alternative and very useful form of this expression is obtained by adding and subtracting $\Delta(B)\Sigma\, \lambda_k$ to (1) to get:

$$\varphi(B) = \sum_{k=1}^{p} \lambda_k \Delta(B)\, (1-\delta_k(B)) / \delta_k(B) + \Delta(B)\, \varphi^{*}(B), \tag{6}$$

where $\varphi^{*}(B) = \varphi^{**}(B) + \Sigma\, \lambda_k$. In this expression, $\varphi(0) = \varphi^{*}(0)$ which is normalized to unity.

It is clear that the polynomial $\varphi(B)$ will have a root at θ_k if and only if $\lambda_k = 0$. Thus, testing for unit roots can be carried out equivalently by testing for parameters $\lambda_k = 0$ is an appropriate expansion.

In our case, we try to test the existence of the seasonal unit roots for the monthly time series. So, our equation:

$$y_t = y_{t-12} + \varepsilon_t,$$

$$(1-B^{12})y_t = \varepsilon_t. \tag{7}$$

To find out roots of the (7), first try to factorize the $1-B^{12}$:

$$1-B^{12} = 0, \quad B^{12} = 1 \tag{8}$$

$$B^{12} = 1^{1/12}\exp(\, i2\pi k/n\,) \qquad k = 0, \ldots, 11 \qquad n = 12$$

As a result, 12 roots of equation (8) are as follows:

$$1-B^{12} = (1-B_1)(1-B_2)(1-B_3)(1-B_4)(1-B_5)(1-B_6)(1-B_7)(1-B_8)(1-B_9)(1-B_{10})(1-B_{11})(1-B_{12})$$

where:

$$B_1 = 1,$$

$B_2 = 1.\exp(i12\pi/12) = \cos(\pi) + i\sin(\pi) = -1,$

$B_3 = 1.\exp(i18\pi/12) = \cos(3\pi/2) + i\sin(3\pi/2) = -i,$

$B_4 = 1.\exp(i6\pi/12) = \cos(\pi/2) + i\sin(\pi/2) = i,$

$B_5 = 1.\exp(i10\pi/12) = \cos(5\pi/6) + i\sin(5\pi/6) = -(\surd 3)/2 + i(1/2),$

$B_6 = 1.\exp(i14\pi/12) = \cos(7\pi/6) + i\sin(7\pi/6) = -(\surd 3)/2 - i(1/2),$

$B_7 = 1.\exp(i22\pi/12) = \cos(11\pi/6) + i\sin(11\pi/6) = (\surd 3)/2 - i(1/2),$

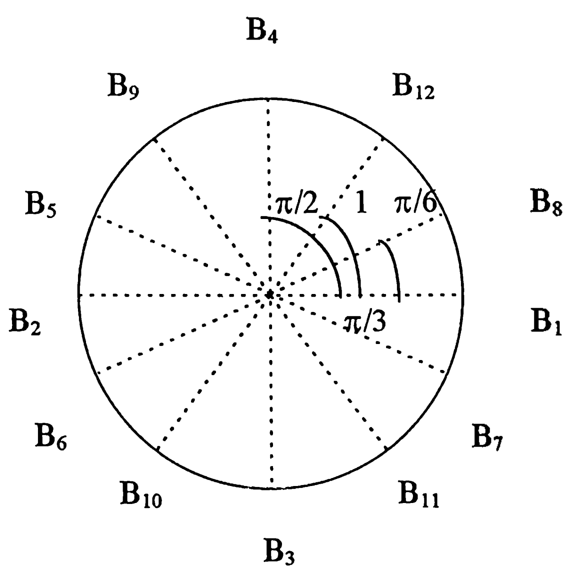
$B_8 = 1.\exp(i2\pi/12) = \cos(\pi/6) + i\sin(\pi/6) = (\surd 3)/2 + i(1/2),$

$B_9 = 1.\exp(i8\pi/12) = \cos(2\pi/3) + i\sin(2\pi/3) = -1/2 + i(\surd 3)/2,$

$B_{10} = 1.\exp(i16\pi/12) = \cos(4\pi/3) + i\sin(4\pi/3) = -1/2 - i(\surd 3)/2,$

$B_{11} = 1.\exp(i20\pi/12) = \cos(5\pi/3) + i\sin(5\pi/3) = 1/2 - i(\surd 3)/2,$

$B_{12} = 1.\exp(i4\pi/12) = \cos(\pi/3) + i\sin(\pi/3) = 1/2 + i(\surd 3)/2.$ (9)



Having applied above proposition testing for the seasonal unit roots in monthly data, expand a polynomial φ(B) about the roots that are found above. Then from (6);

$$\varphi(B) = \lambda_1 (1 + B)(1 + B^2)(1 + B^4 + B^8)(B) + \lambda_2 (1 - B)(1 + B^2)(1 + B^4 + B^8)(-B)$$

$$+ \lambda_3 (1 + iB)(1 - B^2)(1 + B^4 + B^8)(iB) + \lambda_4 (1 - iB)(1 - B^2)(1 + B^4 + B^8)(-iB)$$

$$+ \lambda_5 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 - \sqrt{3}B + B^2)(1 + ((\sqrt{3} - i)B/2))$$

$$(-(\sqrt{3} + i)B/2)$$

$$+ \lambda_6 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 - \sqrt{3}B + B^2)(1 + ((\sqrt{3} + i)B/2))$$

$$(-(\sqrt{3} - i)B/2)$$

$$+ \lambda_7 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 + \sqrt{3}B + B^2)(1 - ((\sqrt{3} - i)B/2))$$

$$((\sqrt{3} + i)B/2)$$

$$+ \lambda_8 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 + \sqrt{3}B + B^2)(1 - ((\sqrt{3} + i)B/2))$$

$$((\sqrt{3} - i)B/2)$$

$$+ \lambda_9 (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 - B + B^2)(1 - ((i\sqrt{3} - 1)B/2))$$

$$(-(i\sqrt{3} + 1)B/2)$$

$$+ \lambda_{10} (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 - B + B^2)(1 + ((i\sqrt{3} + 1)B/2))$$

$$((i\sqrt{3} - 1)B/2)$$

$$+ \lambda_{11} (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 + B + B^2)(1 + ((i\sqrt{3} - 1)B/2))$$

$$((i\sqrt{3} + 1)B/2)$$

$$+ \lambda_{12} (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 + B + B^2)(1 - ((i\sqrt{3} + 1)B/2))$$

$$(-(i\sqrt{3} - 1)B/2)$$

$$+ \varphi^*(B)(1-B^{12}). \qquad (10)$$

Clearly, λ_3 and λ_4, λ_5 and λ_6, λ_7 and λ_8, λ_9 and λ_{10}, λ_{11} and λ_{12}, must be complex conjugates since $\varphi(B)$ is real. Simplifying and substituting as:

$$\{\lambda_1 = -\pi_1, \lambda_2 = -\pi_2\},$$

$$\{\lambda_3 = (-\pi_4 + i\pi_3)/2, \lambda_4 = (-\pi_4 - i\pi_3)/2\},$$

$$\{\lambda_5 = (-\pi_6 + i\pi_5)/2, \lambda_6 = (-\pi_6 - i\pi_5)/2\},$$

$$\{\lambda_7 = (-\pi_8 + i\pi_7)/2, \lambda_8 = (-\pi_8 - i\pi_7)/2\},$$

$$\{\lambda_9 = (-\pi_{10} + i\pi_9)/2, \lambda_{10} = (-\pi_{10} - i\pi_9)/2\}.$$

$$\{\lambda_{11} = (-\pi_{12} + i\pi_{11})/2, \lambda_{12} = (-\pi_{12} - i\pi_{11})/2\}. \tag{11}$$

then (10) will be obtained as,

$$\begin{aligned}\varphi(B) = &-\pi_1(1+B)(1+B^2)(1+B^4+B^8) - \pi_2(-(1-B)(1+B^2)(1+B^4+B^8))\\ &-(\pi_3 + \pi_4 B)(-(1-B^2)(1+B^4+B^8))\\ &-(\pi_5 + \pi_6 B)(-(1-B^4)(1-\sqrt{3}B+B^2)(1+B^2+B^4))\\ &-(\pi_7 + \pi_8 B)(-(1-B^4)(1+\sqrt{3}B+B^2)(1+B^2+B^4))\\ &-(\pi_9 + \pi_{10} B)(-(1-B^4)(1-B^2+B^4)(1-B+B^2))\\ &-(\pi_{11} - \pi_{12} B)(-(1-B^4)(1-B^2+B^4)(1+B+B^2))\\ &+\varphi^*(B)(1-B^{12})\end{aligned} \tag{12}$$

The testing strategy is now apparent. The data are assumed to be generated by a general autoregression;

$$\varphi(B)y_t = \varepsilon_t, \tag{13}$$

And (12) is used to replace $\varphi(B)$, giving:

$$\varphi^*(B)y_{8,t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-1} + \pi_4 y_{3,t-2} + \pi_5 y_{4,t-1} + \pi_6 y_{4,t-2} + \pi_7 y_{5,t-1} + \pi_8 y_{6,t-2}$$
$$+ \pi_9 y_{6,t-1} + \pi_{10} y_{6,t-2} + \pi_{11} y_{7,t-1} + \pi_{12} y_{7,t-2} + \mu_t + \varepsilon_t \qquad (14)$$

where:

$$y_{1,t} = (1 + B)(1 + B^2)(1 + B^4 + B^8)y_t,$$
$$y_{2,t} = -(1 - B)(1 + B^2)(1 + B^4 + B^8)y_t,$$
$$y_{3,t} = -(1 - B^2)(1 + B^4 + B^8)y_t,$$
$$y_{4,t} = -(1 - B^4)(1 - \sqrt{3}B + B^2)(1 + B^2 + B^4)y_t,$$
$$y_{5,t} = -(1 - B^4)(1 + \sqrt{3}B + B^2)(1 + B^2 + B^4)y_t,$$
$$y_{6,t} = -(1 - B^4)(1 - B^2 + B^4)(1 - B + B^2)y_t,$$
$$y_{7,t} = -(1 - B^4)(1 - B^2 + B^4)(1 + B + B^2)y_t,$$
$$y_{8,t} = (1-B^{12})\, y_t = \Delta_{12} y_t. \qquad (15)$$

Testing for unit roots in monthly time series is equivalent to testing for the significance of the parameters in the auxiliary regression where $\varphi^*(B)$ is some polynomial function of B for which the usual assumption applies.

To test the hypothesis that $\Psi(\theta_k) = 0$, where θ_k is either of the roots of equation (10). one needs simply to test that λ_k is zero. For the root 1, this simply a test for $\pi_1=0$, and for -1 it's $\pi_2=0$. For the complex roots λ_3 will have absolute value of zero only if both π_3 and π_4 equal to zero, which suggest a joint test. There will be no seasonal unit roots if π_2 and either π_3 or π_4 are different from the zero, which therefore requires the rejection of both a test for π_2 and a joint test for π_3 and π_4. To find that a series has no unit roots at all and is therefore stationary, we must establish that each of the π's is different from zero (save possibility either π_3 or π_4). A joint test will not deliver the required evidence. I have to note that, the same arguments for π_3 and π_4 are also applicable for π_5 and π_6 , π_7 or π_8 , π_9 and π_{10}, and π_{11} and π_{12} . So the joint test that is useful for π_3 and π_4 are useful for π_5 and π_6 , π_7 or π_8 , π_9 and π_{10}, and π_{11} and π_{12} .

The natural alternative for these tests is stationarity. For example, the alternative to $\varphi(1) = 0$ should be $\varphi(1) > 0$ which means $\pi_1 < 0$. Similarly, the stationarity alternative to $\varphi(-1) = 0$ is $\varphi(-1) > 0$ which correspondence to $\pi_2 < 0$. The alternative for complex root at i is to $|\varphi(i)| = 0$ is $|\varphi(i)| > 0$. Since the null space of π_1 and π_2 is two dimensional, it is simplest to compute an F-type of statistic for the joint null, $\pi_3 = \pi_4 = 0$, against the alternative that they are not both equal to zero. An alternative strategy is to compute a two-sided test of $\pi_4 = 0$, and if this is accepted, continue with a one-sided test of $\pi_3 = 0$ against the alternative $\pi_3 < 0$. If we restrict our attention to alternatives where it is assumed that $\pi_4 = 0$, a one sided test for π_3 would be appropriate with rejection for $\pi_3 < 0$. Potentially, this could lack power if the first-step assumption is not warranted. Here, also I have to note that, the same arguments for π_3 and π_4 are also applicable for π_5 and π_6 , π_7 or π_8 , π_9 and π_{10}, and π_{11} and π_{12} .

If some of the π's are zero, there are other unit roots in the regression. However, as we know, y_i's are asymptotically uncorrelated. The distribution of the test statistic will not be affected by the inclusion of a variable with a zero coefficient which is orthogonal to the included variables. For example, when testing $\pi_1 = 0$, suppose $\pi_2 = 0$ but y_{2t} is still included in the regression. Then y_{1t} and y_{2t} will be asymptotically uncorrelated with lags of y_{12t} which is stationary. The test for $\pi_1 = 0$ will have the same limiting distribution regardless of whether y_{2t} is included in the regression. Similar arguments follow for the other cases, also.

2.4 INFLUENCES OF DETERMINISTIC COMPONENTS TO THE TEST

Applying ordinary least squares to equation (14) gives estimates of the π_i . In case, there are seasonal unit roots, corresponding π_i are zero. Due to the fact that pairs of complex unit roots are conjugates, it should be noted that these roots are only present when pairs of π's are equal to zero simultaneously, for example the roots i and -i are only present when π_3 and π_4 are equal to zero. There will be no seasonal unit roots if π_2 through π_{12} are significantly different from zero. If $\pi_2 = 0$, then the presence of root -1 can not be rejected. When $\pi_1 = 0$, π_2 through π_{12} are unequal to zero, and when, additionally, seasonality can be modeled with seasonal dummies.

In the more complex setting where the alternative includes the possibility of deterministic components, it is necessary to allow $\mu_t \neq 0$. The testable model becomes:

$$\varphi^*(B)y_{8,t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-1} + \pi_4 y_{3,t-2} + \pi_5 y_{4,t-1} + \pi_6 y_{4,t-2} + \pi_7 y_{5,t-1} + \pi_8 y_{6,t-2}$$

$$+ \pi_9 y_{6,t-1} + \pi_{10} y_{6,t-2} + \pi_{11} y_{7,t-1} + \pi_{12} y_{7,t-2} + \mu_t + \varepsilon_t$$

which can again be estimated by OLS and the statistics on the π's used for interface.

When deterministic components (like constant, seasonal dummies, and trend) are present in the regression the distributions change. Again, the changes can be anticipated from this general approach. The intercept and trend portions of the deterministic mean influence only the distribution of π_1 because they have all their spectral mass at zero frequency. Once the intercept is included, the remaining eleven seasonal dummies do not affect the limiting distribution of π_1 . The seasonal dummies, however, do affect the distribution of π_2, π_3..., π_{12}.

CHAPTER 3

3. SEASONAL COINTEGRATION AND ERROR CORRECTION REPRESENTATION FOR MONTHLY SERIES

3.1 DEFINITION

The theory and application of cointegration have been of major interest in economics for a number of years. Recently the theory was extended to cover aspects of economic time series other than the long-run or the zero frequency characteristics. Especially, Hylleberg-Engle-Granger-Yoo (HEGY) (1990) consider the seasonal frequency in quarterly time series.

Based on the definition of integration at a specific frequency, HEGY (1990) extend the theory of cointegrated systems to cover cointegration at frequencies other than the long-run frequency. Let us consider an N x 1 vector of zero mean variables y_t which are all I(1) at the frequencies $\theta = 0, \pi, 3\pi/2, \pi/2, 5\pi/6, 7\pi/6, 11\pi/6, \pi/6, 2\pi/3, 4\pi/3, 5\pi/3, \pi/3$. Again, the Wold representation can then be written as:

$$(1-B^{12})y_t = C(B)\varepsilon_t$$

where ε_t is an N x 1 vector of n.i.d. $(0, \Omega)$ variables and C(B) are N x N matrix of lag polynomials.

Granger (1981) proposed the concept of cointegration which recognized that even though several series all had unit roots, some linear combination of them could not have unit root at all.

A pair of series each of which are integrated at frequency w are said to be cointegrated at that frequency if a linear combination of the series is not integrated at w. If the linear combination is labeled α, then we use the notation:

$$x_t \sim CI_w$$

with cointegrating vector α. This will occur if, for example, each of the series contains the same factor which is $I_w(1)$. In particular, if:

$$x_t = \alpha v_t + \underline{x}_t \text{ and } y_t = v_t + \underline{y}_t,$$

where v_t is $I_w(1)$ and $\underline{x}_t$ and $\underline{y}_t$ are not, then $z_t = x_t - \alpha y_t$ is not $I_w(1)$, although it could be still integrated at other frequencies, if a group of series are cointegrated.

Cointegration at the zero frequency then depends on the existence of an N x r matrix α_1, $N > r_1 \geq 0$ such that $\alpha_1' C(1) = 0$, while the cointegration at the frequency 1/2 requires the existence of an N x r_2 matrix α_2 such that $\alpha_2' C(-1) = 0$. The columns in α_1 and α_2 are called the cointegrating vectors at the frequencies 0 and 1/2, respectively, while r_1

and r_2 are called the cointegrating ranks. Cointegration at the frequencies $3\pi/2$, $\pi/2, 5\pi/6$, $7\pi/6$, $11\pi/6$, $\pi/6$, $2\pi/3$, $4\pi/3$ corresponding to other roots as shown in equation (9).

$$s = \{ (-\sqrt{3}/2 + i/2), (-\sqrt{3}/2 - i/2), (\sqrt{3}/2 - i/2), (\sqrt{3}/2 + i/2), (i\sqrt{3}/2 - 1/2), (-i\sqrt{3}/2 - 1/2), (-i\sqrt{3}/2 + 1/2), (i\sqrt{3}/2 + 1/2)\}$$

are most elegantly handled by extending the notion of a cointegrating vector to that of a cointegrating polynomial vector in $\alpha(B) = \alpha_m + \alpha_{m+1}B$ such that $\alpha'(s)C(s) = 0$, where α_m and α_{m+1} are N x r_m vectors, $N > r_m \geq 0$, and where m = 3, ..., 11

3.2 TESTING PROCEDURE

Least square regression will give a superconsistent estimate of the cointegration parameters as in the Engle and Granger two step method. Furthermore, these estimates can be used directly in specifying and estimating the error correction model, and tests for cointegration at these frequencies can be carried out by testing the residuals from such cointegrating regressions for any remaining unit roots at the particular frequencies.

Let, y_t be an N x 1 vector of monthly time series, each of which potentially has unit roots at zero and all seasonal frequencies, so that each component of $(1-B^{12})y_t$ is stationary process but may have a zero on the unit circle. Again, the Wold representation will thus be;

$$(1-B^{12})y_t = C(B)\varepsilon_t,$$

where ε_t is a vector white noise process with zero mean and covariance matrix Ω, a positive definite matrix.

There are a variety of possible types of cointegration for such a set of series. To initially examine these, apply the decomposition of (1) to each element of C(B). This gives:

$$C(B) = \sum_{k=1}^{p} \Lambda_k \, \Delta(B) / \delta_k(B) + C^{**}(B) \, \Delta(B), \tag{16}$$

where $\delta_k(B) = 1 - (1 / \theta_k)B$ and $\Delta(B)$ is the product of all the $\delta_k(B)$ as shown at equation (3). For monthly data, the twelve roots, θ_k's, are given in equation (9) that solving for the Λ's becomes:

$$\begin{aligned}
C(B) = {} & \varsigma_1 (1 + B)(1 + B^2)(1 + B^4 + B^8) + \varsigma_2 (1 - B)(1 + B^2)(1 + B^4 + B^8) \\
& + \varsigma_3 (1 + iB)(1 - B^2)(1 + B^4 + B^8) + \varsigma_4 (1 - iB)(1 - B^2)(1 + B^4 + B^8) \\
& + \varsigma_5 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 - \sqrt{3}B + B^2)(1 + ((\sqrt{3} - i)B/2)) \\
& + \varsigma_6 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 - \sqrt{3}B + B^2)(1 + ((\sqrt{3} + i)B/2)) \\
& + \varsigma_7 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 + \sqrt{3}B + B^2)(1 - ((\sqrt{3} - i)B/2)) \\
& + \varsigma_8 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 + \sqrt{3}B + B^2)(1 - ((\sqrt{3} + i)B/2))
\end{aligned}$$

$$+ \varsigma_9 (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 - B + B^2)(1 - ((i\sqrt{3} - 1)B/2))$$

$$+ \varsigma_{10} (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 - B + B^2)(1 + ((i\sqrt{3} + 1)B/2))$$

$$+ \varsigma_{11} (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 + B + B^2)(1 + ((i\sqrt{3} - 1)B/2))$$

$$+ \varsigma_{12} (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 + B + B^2)(1 - ((i\sqrt{3} + 1)B/2))$$

$$+ C^{**}(B)(1-B^{12}). \tag{17}$$

and if we do the same substitution as we did before, with the same reason:

$$\{\varsigma_1 = -\pi_1 , \varsigma_2 = -\pi_2 \}, \{\varsigma_3 = (-\pi_4 + i\pi_3)/2 , \varsigma_4 = (-\pi_4 - i\pi_3)/2 \},$$

$$\{\varsigma_5 = (-\pi_6 + i\pi_5)/2, \varsigma_6 = (-\pi_6 - i\pi_5)/2 \},$$

$$\{\varsigma_7 = (-\pi_8 + i\pi_7)/2 , \varsigma_8 = (-\pi_8 - i\pi_7)/2\},$$

$$\{\varsigma_9 = (-\pi_{10} + i\pi_9)/2 , \varsigma_{10} = (-\pi_{10} - i\pi_9)/2\},$$

$$\{\varsigma_{11} = (-\pi_{12} + i\pi_{11})/2 , \varsigma_{12} = (-\pi_{12} - i\pi_{11})/2\}.$$

We obtain:

$$\varphi(B) = -\pi_1(1 + B)(1 + B^2)(1 + B^4 + B^8) - \pi_2(-(1 - B)(1 + B^2)(1 + B^4 + B^8))$$

$$- (\pi_3+\pi_4 B)(-(1 - B^2)(1 + B^4 + B^8))$$

$$- (\pi_5+\pi_6 B)(-(1 - B^4)(1 - \sqrt{3}B + B^2)(1 + B^2 + B^4))$$

$$- (\pi_7+\pi_8 B)(-(1 - B^4)(1 + \sqrt{3}B + B^2)(1 + B^2 + B^4))$$

$$- (\pi_9+\pi_{10} B)(-(1 - B^4)(1 - B^2 + B^4)(1 - B + B^2))$$

$$- (\pi_{11}+\pi_{12} B)(-(1 - B^4)(1 - B^2 + B^4)(1 + B + B^2))$$

$$- \varphi^*(B)(1-B^{12}) \tag{18}$$

If we do the same substitution as we did before, equations (18) are obtained :

where:

$$\pi_1 = C(1) / 12,$$

$$\pi_2 = C(-1) / 12,$$

$$\pi_3 = \mathrm{Re}\{C(i)\}/6,\ \pi_4 = \mathrm{Im}\{C(i)\}/6,$$

$$\pi_5 = \mathrm{Re}\{C(-\sqrt{3}/2 + i/2)\}/6,\ \pi_6 = \mathrm{Im}\{C(-\sqrt{3}/2 + i/2)\}/6,$$

$$\pi_7 = \mathrm{Re}\{C(\sqrt{3}/2 - i/2)\}/6,\ \pi_8 = \mathrm{Im}\{C(\sqrt{3}/2 - i/2)\}/6,$$

$$\pi_9 = \mathrm{Re}\{C(i\sqrt{3}/2 - 1/2)\}/6,\ \pi_{10} = \mathrm{Im}\{C(i\sqrt{3}/2 - 1/2)\}/6,$$

$$\pi_{11} = \mathrm{Re}\{C(-i\sqrt{3}/2 + 1/2)\}/6,\ \pi_{12} = \mathrm{Im}\{C(-i\sqrt{3}/2 + 1/2)\}/6, \tag{19}$$

and where:

$$y_{1,t} = (1 + B)(1 + B^2)(1 + B^4 + B^8)y_t,$$

$$y_{2,t} = -(1 - B)(1 + B^2)(1 + B^4 + B^8)y_t,$$

$$y_{3,t} = -(1 - B^2)(1 + B^4 + B^8)y_t,$$

$$y_{4,t} = -(1 - B^4)(1 - \sqrt{3}B + B^2)(1 + B^2 + B^4)y_t,$$

$$y_{5,t} = -(1 - B^4)(1 + \sqrt{3}B + B^2)(1 + B^2 + B^4)y_t,$$

$$y_{6,t} = -(1 - B^4)(1 - B^2 + B^4)(1 - B + B^2)y_t,$$

$y_{7,t} = -(1 - B^4)(1 - B^2 + B^4)(1 + B + B^2)y_t$,

$y_{8,t} = (1-B^{12})\, y_t = \Delta_{12} y_t$.

Multiplying the Wold representation by a vector ξ' gives:

$$(1-B^{12})\, \xi' y_t = \xi' C(B)\, \varepsilon_t \tag{20}$$

Suppose for some $\xi = \xi_1$, $\xi_1' C(1) = 0 = \xi_1' \varsigma_1$, then, there is a factor of (1-B) in all terms, which will cancel out giving;

$$\begin{aligned}
(1+B^4+B^8)(1+B^2)(1+B)\, \xi_1' y_t = \xi_1' \{ & \varsigma_2 (1 + B^2)(1 + B^4 + B^8) \\
& + \varsigma_3 (1 + iB)(1 + B)(1 + B^4 + B^8) + \varsigma_4 (1 - iB)(1 + B)(1 + B^4 + B^8) \\
& + \varsigma_5 (1 + B^2)(1 + B)(1 + B^2 + B^4)(1 - \surd 3B + B^2)(1 + ((\surd 3 - i)B/2)) \\
& + \varsigma_6 (1 + B^2)(1 + B)(1 + B^2 + B^4)(1 - \surd 3B + B^2)(1 + ((\surd 3 + i)B/2)) \\
& + \varsigma_7 (1 + B^2)(1 + B)(1 + B^2 + B^4)(1 + \surd 3B + B^2)(1 - ((\surd 3 - i)B/2)) \\
& + \varsigma_8 (1 + B^2)(1 + B)(1 + B^2 + B^4)(1 + \surd 3B + B^2)(1 - ((\surd 3 + i)B/2)) \\
& + \varsigma_9 (1 + B^2)(1 + B)(1 - B^2 + B^4)(1 - B + B^2)(1 - ((i\surd 3 - 1)B/2)) \\
& + \varsigma_{10} (1 + B^2)(1 + B)(1 - B^2 + B^4)(1 - B + B^2)(1 + ((i\surd 3 + 1)B/2)) \\
& + \varsigma_{11} (1 + B^2)(1 + B)(1 - B^2 + B^4)(1 + B + B^2)(1 + ((i\surd 3 - 1)B/2)) \\
& + \varsigma_{12} (1 + B^2)(1 + B)(1 - B^2 + B^4)(1 + B + B^2)(1 - ((i\surd 3 + 1)B/2)) \\
& + C^{**}(B)\, (1+B^4+B^8)(1+B^2)(1+B) \}\, \varepsilon_t.
\end{aligned} \tag{21}$$

so that $\xi_1' y_t$ will have unit roots at the seasonal frequencies but not at zero frequency. Thus, y is cointegrated at zero frequency with cointegrating vector ξ_1, if $\xi_1' C(1) = 0$. Denote these as:

$y_t \sim CI_0$ with the cointegrating vector ξ_1.

Notice that the vector $y_{1,t} = (1+B^4+B^8)(1+B^2)(1+B)y_t$ is I(1) since $(1-B)y_{1,t} = C(B)\varepsilon_t$, while $\xi_1' y_{1,t}$ is stationary whenever $\xi_1' C(1) = 0$ so that $y_{1,t}$ is cointegrated in the sense that is described by Engle and Granger (1987).

Similarly, letting $y_{2,t} = -(1 - B)(1 + B^2)(1 + B^4 + B^8)y_t$, $(1+B)\, y_{2,t} = -C(B)\, \varepsilon_t$ so that $y_{2,t}$ has a unit root at -1. If $\xi_2' C(-1) = 0$, then $\xi_2' \varsigma_2 = 0$ and $\xi_2' y_{2,t}$ will not have a unit root at -1. We say then that y_t is cointegrated at frequency w = 1/2, which is denoted as:

$y_t \sim CI_{1/2}$ with the cointegrating vector ξ_2.

If x_t has n components, then there may be more than one cointegrating vector ξ. It is clearly possible for several equilibrium relations to govern the joint behavior of the variables.

And, denote $y_{3,t} = -(1 - B^2)(1 + B^4 + B^8)y_t$, which satisfies $(1+B^2)\, y_{3,t} = -C(B)\, \varepsilon_t$ and therefore includes unit roots at frequency 1/4. If $\xi_3' C(i) = 0$, which implies that $\xi_3' \varsigma_3 = \xi_3' \varsigma_4 = 0$, then $\xi_3' y_{3,t}$ will not have a unit root at 1/4, implying that:

$y_t \sim CI_{1/4}$ with the cointegrating vector ξ_3.

We can apply all these procedures to other roots which are located at other frequencies. There is no guarantee that y_t will have any type of cointegration or that these cointegrating vectors will be the same. It is however possible that these cointegrating vectors $\xi_1 = \xi_2 = \xi_3 = \xi_4 = \xi_5 = \xi_6 = \xi_7 = \xi_8 = \xi_9 = \xi_{10} = \xi_{11} = \xi_{12}$, and therefore one cointegrating vector could reduce the integration of the y series at all frequencies. Similarly, if $\xi_2 = \xi_3 = \xi_4 = \xi_5 = \xi_6 = \xi_7 = \xi_8 = \xi_9 = \xi_{10} = \xi_{11} = \xi_{12}$ one cointegrating vector will eliminate the seasonal unit roots. This might be expected if seasonality in the two series is due to the same source.

A characterization of the cointegrating possibilities has now been given in terms of the moving-average representation. More useful are the autoregressive representations and in particular, the error-correction representation. Therefore, if C(B) is a rational matrix in B, it can be written as follows:

$$C(B) = U(B)^{-1}M(B)V(B)^{-1} \qquad (22)$$

where M(B) is a diagonal matrix whose determinants has roots only on the unit circle, and the roots of the determinants of $U(B)^{-1}$ and $V(B)^{-1}$ lie outside the unit circle. This diagonal could contain various combinations of the unit roots. However, assuming that the cointegrating rank at each frequency is r, the matrix can be written as without loss of generality as:

$$M(B) = \begin{vmatrix} I_{N-r} & 0 \\ 0 & \Delta_{12} I_r \end{vmatrix} \tag{23}$$

where I_k is a k X k unit matrix. The following derivation of the error-correction representation is easily adapted for other forms of M(B).

Substituting (23) into Wold Representation and multiplying by U(B) gives:

$$\Delta_{12} U(B) y_t = M(B) V(B)^{-1} \varepsilon_t \tag{24}$$

The first N-r equations have a Δ_{12} on the left side only while the final r equations have Δ_{12} on both sides which therefore cancel out. Thus (24) can be written as:

$$\underline{M(B)} U(B) y_t = V(B)^{-1} \varepsilon_t, \tag{25}$$

with:

$$\underline{M(B)} = \begin{vmatrix} \Delta_{12} I_{N-r} & 0 \\ 0 & I_r \end{vmatrix}$$

Finally, the autoregressive representation is obtained by multiplying by V(B) to obtain;

$$A(B)y_t = \varepsilon_t, \tag{26}$$

where:

$$A(B) = V(B)\underline{M(B)}U(B). \tag{27}$$

Notice that the seasonal and zero-frequency roots, det[$A(\theta)$] = 0 since A(B) has rank r at those frequencies. Now, partition U(B) and V(B) as:

$$U(B) = \begin{vmatrix} U_1(B)' \\ \alpha(B)' \end{vmatrix}, \quad V(B) = [V_1(B), \gamma(B)] \tag{28}$$

where $\alpha(B)$ and $\gamma(B)$ are N X r matrices and $U_1(B)$ and $V_1(B)$ are N X (N - r) matrices.

Expanding the autoregressive matrix using (6) gives:

$$\begin{aligned} A(B) = {} & \lambda_1 (1 + B)(1 + B^2)(1 + B^4 + B^8)(B) + \lambda_2 (1 - B)(1 + B^2)(1 + B^4 + B^8)(-B) \\ & + \lambda_3 (1 + iB)(1 - B^2)(1 + B^4 + B^8)(iB) + \lambda_4 (1 - iB)(1 - B^2)(1 + B^4 + B^8)(-iB) \\ & + \lambda_5 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 - \sqrt{3}B + B^2)(1 + ((\sqrt{3} - i)B/2)) \\ & (-(\sqrt{3} + i)B/2) \end{aligned}$$

$$+ \lambda_6 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 - \sqrt{3}B + B^2)(1 + ((\sqrt{3} + i)B/2))$$

$$(-(\sqrt{3} - i)B/2)$$

$$+ \lambda_7 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 + \sqrt{3}B + B^2)(1 - ((\sqrt{3} - i)B/2))$$

$$((\sqrt{3} + i)B/2)$$

$$+ \lambda_8 (1 + B^2)(1 - B^2)(1 + B^2 + B^4)(1 + \sqrt{3}B + B^2)(1 - ((\sqrt{3} + i)B/2))$$

$$((\sqrt{3} - i)B/2)$$

$$+ \lambda_9 (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 - B + B^2)(1 - ((i\sqrt{3} - 1)B/2))$$

$$(-(i\sqrt{3} + 1)B/2)$$

$$+ \lambda_{10} (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 - B + B^2)(1 + ((i\sqrt{3} + 1)B/2))$$

$$((i\sqrt{3} - 1)B/2)$$

$$+ \lambda_{11} (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 + B + B^2)(1 + ((i\sqrt{3} - 1)B/2))$$

$$((i\sqrt{3} + 1)B/2)$$

$$+ \lambda_{12} (1 + B^2)(1 - B^2)(1 - B^2 + B^4)(1 + B + B^2)(1 - ((i\sqrt{3} + 1)B/2))$$

$$(-(i\sqrt{3} - 1)B/2)$$

$$+ \varphi^*(B)(1-B^{12}). \tag{29}$$

By applying the same procedure, we obtain;

$$A(B) = -\pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} - (\pi_3+\pi_4 B)y_{3,t-1} - (\pi_5+\pi_6 B)y_{4,t-1} - (\pi_7+\pi_8 B)y_{5,t-1}$$

$$- (\pi_9+\pi_{10} B)y_{6,t-1} - (\pi_{11}+\pi_{12} B)y_{7,t-1} + A^*(B)(1-B^{12}) \tag{30}$$

where:

$\pi_1 = -\gamma(1)\alpha'(1)/12 = -\gamma_1\alpha'_1,$ where $\gamma(1) = \gamma_1$ and $\alpha'(1)/12 = \alpha_1,$

$\pi_2 = -\gamma(-1)\alpha'(-1)/12 = -\gamma_2\alpha'_2,$ where $\gamma(-1) = \gamma_2$ and $\alpha'(-1)/12 = \alpha_2,$

$\pi_3 = \mathrm{Re}\{\gamma(i)\alpha'(i)\}/6,$ where $\mathrm{Re}\{\gamma(i)\} = \gamma_3$ and $\mathrm{Re}\{\alpha(i)/6\} = \alpha_3,$

$\pi_4 = \mathrm{Im}\{\gamma(i)\alpha'(i)\}/6,$ where $\mathrm{Im}\{\gamma(i)\} = \gamma_4$ and $\mathrm{Im}\{\alpha(i)/6\} = \alpha_4,$

$\pi_5 = \mathrm{Re}\{\gamma(-\sqrt{3}/2 + i/2)\, \alpha'(-\sqrt{3}/2 + i/2)\}/6$

where $\mathrm{Re}\{\gamma\, (-\sqrt{3}/2 + i/2)\} = \gamma_5$ and $\mathrm{Re}\{\alpha(-\sqrt{3}/2 + i/2)/6\} = \alpha_5,$

$\pi_6 = \mathrm{Im}\{\gamma(-\sqrt{3}/2 + i/2)\, \alpha'(-\sqrt{3}/2 + i/2)\}/6$

where $\mathrm{Im}\{\gamma\, (-\sqrt{3}/2 + i/2)\} = \gamma_6$ and $\mathrm{Im}\{\alpha(-\sqrt{3}/2 + i/2)/6\} = \alpha_6,$

$\pi_7 = \mathrm{Re}\{\gamma(\sqrt{3}/2 - i/2)\, \alpha'(\sqrt{3}/2 - i/2)\}/6$

where $\mathrm{Re}\{\gamma\, (\sqrt{3}/2 - i/2)\} = \gamma_7$ and $\mathrm{Re}\{\alpha(\sqrt{3}/2 - i/2)/6\} = \alpha_7,$

$\pi_8 = \mathrm{Im}\{\gamma(\sqrt{3}/2 - i/2)\, \alpha'(\sqrt{3}/2 - I/2)\}/6$

where $\mathrm{Im}\{\gamma\, (\sqrt{3}/2 - i/2)\} = \gamma_8$ and $\mathrm{Im}\{\alpha(\sqrt{3}/2 - i/2)/6\} = \alpha_8,$

$\pi_9 = \mathrm{Re}\{\gamma(i\sqrt{3}/2 - 1/2)\, \alpha'(i\sqrt{3}/2 - 1/2)\}/6$

where $\mathrm{Re}\{\gamma\, (i\sqrt{3}/2 - 1/2)\} = \gamma_9$ and $\mathrm{Re}\{\alpha(i\sqrt{3}/2 - 1/2)/6\} = \alpha_9,$

$\pi_{10} = \mathrm{Im}\{\gamma(i\sqrt{3}/2 - 1/2)\, \alpha'(i\sqrt{3}/2 - 1/2)\}/6$

where $\mathrm{Im}\{\gamma\, (i\sqrt{3}/2 - 1/2)\} = \gamma_{10}$ and $\mathrm{Im}\{\alpha(i\sqrt{3}/2 - 1/2)/6\} = \alpha_{10},$

$\pi_{11} = \mathrm{Re}\{\gamma(-i\sqrt{3}/2 + 1/2)\, \alpha'(-i\sqrt{3}/2 + 1/2)\}/6$

where $\mathrm{Re}\{\gamma\, (-i\sqrt{3}/2 + 1/2)\} = \gamma_{11}$ and $\mathrm{Re}\{\alpha(-i\sqrt{3}/2 + 1/2)/6\} = \alpha_{11},$

$\pi_{12} = \mathrm{Im}\{\gamma(-i\sqrt{3}/2 + 1/2)\, \alpha'(-i\sqrt{3}/2 + 1/2)\}/6$

where $\mathrm{Im}\{\gamma\, (-i\sqrt{3}/2 + 1/2)\} = \gamma_{12}$ and $\mathrm{Im}\{\alpha(-i\sqrt{3}/2 + 1/2)/6\} = \alpha_{12},$ (31)

CHAPTER 4

4. ERROR CORRECTION

4.1 DEFINITION

Error correction mechanism have been used widely in economics. The idea is simply that a proportion of the disequilibrium from one period is corrected in the next period. For example, the change in price in one period may depend upon the degree of excess demand in the previous period. Such schemes can be derived as optimal behavior with some types of adjustment costs or incomplete information.

For a two variables system a typical error correction model would relate the change in one variable to past equilibrium errors, as well as to past changes in both variables. For a multivariate system we can define a general error correction representation in terms of B, the backshift operator.

In this section, an error-correction representation is derived which explicitly takes the cointegration restrictions at the zero and at the seasonal frequencies into account. As the time series being considered, it has poles at different locations on the unit circle, and various cointegrating situations are possible. This naturally makes the general treatment mathematically complex. Although, we treat the general case and present the special cases considered to be of most interest.

An individual economic variable, viewed as a time series, can wonder extensively and yet some pairs of series may be expected to move so that they do not drift too far apart. Typically economic theory will propose forces which tend to keep such series together. Examples might be short and long term interest rates, capital appropriations and expenditures, household income and expenditures, and prices of the same commodity in different markets or close substitutes in the same market. A similar idea arises from considering equilibrium is a stationary point characterized by forces which tend to push the economy back toward equilibrium whenever it moves away. If x_t is a vector of economic variables, then they may be said to be in equilibrium when the specific linear constraint:

$$\alpha' x_t = 0$$

occurs. In most time periods, x_t will not be in equilibrium and the univariate quantity:

$$z_t = \alpha' x_t$$

may be called the equilibrium error. If the equilibrium concept is to have any relevance for the specification of econometric models, the economy should appear to prefer a small value of z_t rather than a large value.

4.2 TESTING PROCEDURE

The general error-correction model can be written as:

$$
\begin{aligned}
A^*(B)\Delta_{12}y_t = {} & \gamma_1\alpha_1'y_{1,t-1} + \gamma_2\alpha_2'y_{2,t-1} + (\gamma_4\alpha_3' + \gamma_3\alpha_4')y_{3,t-1} - (\gamma_3\alpha_3' - \gamma_4\alpha_4')y_{3,t-2} \\
& + (\gamma_6\alpha_5' + \gamma_5\alpha_6')y_{4,t-1} - (\gamma_5\alpha_5' - \gamma_6\alpha_6')y_{4,t-2} \\
& + (\gamma_8\alpha_7' + \gamma_7\alpha_8')y_{5,t-1} - (\gamma_7\alpha_7' - \gamma_8\alpha_8')y_{5,t-2} \\
& + (\gamma_{10}\alpha_9' + \gamma_9\alpha_{10}')y_{6,t-1} - (\gamma_9\alpha_9' - \gamma_{10}\alpha_{10}')y_{6,t-2} \\
& + (\gamma_{12}\alpha_{11}' + \gamma_{11}\alpha_{12}')y_{7,t-1} - (\gamma_{11}\alpha_{11}' - \gamma_{12}\alpha_{12}')y_{7,t-2} + \varepsilon_t
\end{aligned}
\tag{32}
$$

where $A^*(0) = C(0) = I_N$ in the standard case. This expression is an error-correction representation where both α, the cointegrating vector, and γ, the coefficients of the error-correction term, may be different lags. This can be written in a more transparent form by allowing more than two lags in the error-correction term. Add:

$$
\sum_{i=3}^{11} \Delta_{12}(\gamma_i\alpha_{i+1}' + \gamma_{i+1}\alpha_i' + \gamma_{i+1}\alpha_{i+1}'B)y_{t-1}
$$

to both sides and rearrange terms to get:

$$
\begin{aligned}
\underline{A^*(B)}\,\Delta_{12}y_t = {} & \gamma_1\alpha_1'y_{1,t-1} + \gamma_2\alpha_2'y_{2,t-1} + (\gamma_4B + \gamma_3)(\alpha_3' + \alpha_4'B)y_{3,t-2} \\
& + (\gamma_6B + \gamma_5)(\alpha_5' + \alpha_6'B)y_{4,t-2} + (\gamma_8B + \gamma_7)(\alpha_7' + \alpha_8'B)y_{5,t-2} \\
& + (\gamma_{10}B + \gamma_9)(\alpha_9' + \alpha_{10}'B)y_{6,t-2} + (\gamma_{12}B + \gamma_{11})(\alpha_{11}' + \alpha_{12}'B)y_{7,t-2} + \varepsilon_t
\end{aligned}
\tag{33}
$$

where γ_1, γ_2, γ_3, γ_4, γ_5, γ_6, γ_7, γ_8, γ_9, γ_{10}, γ_{11}, and γ_{12} are N x r_1, N x r_2, N x r_3, N x r_3, N x r_4, N x r_4, N x r_5, N x r_5, N x r_6, N x r_6, N x r_7, N x r_7 matrices, respectively, where $\underline{A^*(B)}$ is slightly different autoregressive matrix from $A^*(B)$. The two error correction term at the annual and semi-annual seasonal enter with one lag while at the other frequencies, they entered with the two lags (as can be seen from equation (41)) and when $\{\alpha_4,\gamma_4\}$, $\{\alpha_6, \gamma_6\}$, $\{\alpha_8, \gamma_8\}$, $\{\alpha_{10}, \gamma_{10}\}$, and $\{\alpha_{12}, \gamma_{12}\}$ are equal to zero, the model simplifies so that, respectively, cointegration is contemporaneous, the error correction compose of one lag terms.

For the first two a least squares regression will give a superconsistent estimate of cointegration parameters as in the Engle-Granger two-step method. Furthermore, these estimates can be used directly in specifying and estimating the error correction model, and tests for cointegration at these frequencies can be carried out by testing the residuals such cointegrating regressions for any remaining unit roots at the particular frequencies 0 and 1/2.

Engle et. al. (EGHL) (1993) propose a test procedure for the presence of seasonal and nonseasonal cointegration relations. Suppose that two time series x_t and y_t have some or all unit roots at nonseasonal and/or seasonal frequencies. When there is cointegration at the zero frequency, i.e. when x_t and y_t have a common nonseasonal unit root, the process u_t defined by:

$$u_t = (1 + B)(1 + B^2)(1 + B^4 + B^8)x_t - \alpha_1(1 + B)(1 + B^2)(1 + B^4 + B^8)y_t, \qquad (34)$$

is a stationary process. Seasonal cointegration at the bi-annual frequency π, corresponding to unit root -1, amounts to the stationarity of the process v_t, which is defined by:

$$v_t = (1 - B)(1 + B^2)(1 + B^4 + B^8)x_t - \alpha_2 (1 - B)(1 + B^2)(1 + B^4 + B^8)y_t, \qquad (35)$$

Seasonal cointegration at the annual frequency $\pi/2$, corresponding to the unit roots $\pm$ i, amounts to the stationarity of the process w_t, defined by:

$$w_t = (1 - B^2)(1 + B^4 + B^8)x_t - \alpha_3(1 - B^2)(1 + B^4 + B^8)y_t - \alpha_4(1 - B^2)(1 + B^4 + B^8)x_{t-1}$$
$$- \alpha_5(1 - B^2)(1 + B^4 + B^8)y_{t-1}, \qquad (36)$$

And, in other frequencies:

$$a_t = (1 - B^4)(1 - \sqrt{3}B + B^2)(1 + B^2 + B^4)x_t - \alpha_6(1 - B^4)(1 - \sqrt{3}B + B^2)(1 + B^2 + B^4)y_t -$$
$$\alpha_7(1 - B^4)(1 - \sqrt{3}B + B^2)(1 + B^2 + B^4)x_{t-1} - \alpha_8(1 - B^4)(1 - \sqrt{3}B + B^2)(1 + B^2 + B^4)y_{t-1},$$
$$(37)$$

$$b_t = (1 - B^4)(1 + \sqrt{3}B + B^2)(1 + B^2 + B^4)x_t - \alpha_9(1 - B^4)(1 + \sqrt{3}B + B^2)(1 + B^2 + B^4)y_t -$$
$$\alpha_{10}(1 - B^4)(1 + \sqrt{3}B + B^2)(1 + B^2 + B^4)x_{t-1} - \alpha_{11}(1 - B^4)(1 + \sqrt{3}B + B^2)(1 + B^2 + B^4)y_{t-1},$$
$$(38)$$

$$c_t = (1 - B^4)(1 - B^2 + B^4)(1 - B + B^2)x_t - \alpha_{12}(1 - B^4)(1 - B^2 + B^4)(1 - B + B^2)y_t -$$

$$\alpha_{13}(1 - B^4)(1 - B^2 + B^4)(1 - B + B^2)x_{t-1} - \alpha_{14}(1 - B^4)(1 - B^2 + B^4)(1 - B + B^2)y_{t-1}, \quad (39)$$

$$d_t = (1 - B^4)(1 - B^2 + B^4)(1 + B + B^2)x_t - \alpha_{15}(1 - B^4)(1 - B^2 + B^4)(1 + B + B^2)y_t -$$

$$\alpha_{16}(1 - B^4)(1 - B^2 + B^4)(1 + B + B^2)x_{t-1} - \alpha_{17}(1 - B^4)(1 - B^2 + B^4)(1 + B + B^2)y_{t-1}, \quad (40)$$

In case all u_t, v_t, w_t, a_t, b_t, c_t, and d_t series are stationary, a simplified version of the seasonal cointegration model is:

$$\Delta_{12}x_t = \mu + \beta\Delta_{12}y_{t-1} + \gamma_1 u_{t-1} + \gamma_2 v_{t-1} + \gamma_3 w_{t-2} + \gamma_4 w_{t-3} + \gamma_5 a_{t-2} + \gamma_6 a_{t-3} + \gamma_7 b_{t-2} + \gamma_8 b_{t-3}$$

$$+ \gamma_9 c_{t-2} + \gamma_{10} c_{t-3} + \gamma_{11} d_{t-2} + \gamma_{12} d_{t-3}, \quad (41)$$

where μ is an intercept term, and where γ_1 to γ_{12} are adjustment parameters.

The test method proposed in EGHL is a two step method, similar to Engle and Granger's approach to nonseasonal time series. The first step involves the estimation of the α_1 to α_{17} parameters by simple regressions, where such regressions may include a constant, seasonal dummies and a trend if necessary, and a test whether the residual processes $\underline{u}_t$, $\underline{v}_t$, $\underline{w}_t$, $\underline{a}_t$, $\underline{b}_t$, $\underline{c}_t$, and $\underline{d}_t$ are stationary. The second step is to replace the u_t, v_t, w_t, a_t, b_t, c_t, and d_t processes in (41) by their estimated counterparts, and to test the significance of the adjustment parameters. The later step involves standard asymptotics for the t values for the γ_i's while the first step involves (extension of the) Engle and Granger (1987) type asymptotics. For example, to test whether there is nonseasonal cointegration, one checks whether $\rho = 0$ in the auxiliary regression:

$$(1-B)\underline{u}_t = -\rho\, \underline{u}_{t-1} + \sum_{i=1}^{p} \lambda_i(1-B)\underline{u}_{t-i} + \varepsilon_t \qquad (42)$$

The critical values of this so-called Augmented Dickey-Fuller (ADF) t test for ρ are those tabulated in Engle and Granger (1987). Similarly, to test for seasonal cointegration at frequency π, one tests whether $\rho = 0$ in the auxilary regression:

$$(1+B)\underline{v}_t = -\rho\underline{v}_{t-1} + \sum_{i=1}^{p} \lambda_i(1+B)\underline{v}_{t-i} + \varepsilon_t \qquad (43)$$

Similarly, testing for frequencies $\pi/2$, one has to test whether ρ_1 and ρ_2 equal to 0 by using the equation:

$$(1+B^2)\underline{w}_t = -\rho_1\underline{w}_{t-2} - \rho_2\underline{w}_{t-1} + \sum_{i=1}^{p} \lambda_i(1+B^2)\underline{w}_{t-i} + \varepsilon_t \qquad (44)$$

Similarly, testing for frequencies $5\pi/6$, $7\pi/6$, one has to test whether ρ_1 and ρ_2 equal to 0 by using the equation:

$$(1+\sqrt{3}B+B^2)\underline{a}_t = -\rho_1\underline{a}_{t-2} - \rho_2\underline{a}_{t-1} + \sum_{i=1}^{p} \lambda_i(1+\sqrt{3}B+B^2)\underline{a}_{t-i} + \varepsilon_t \qquad (45)$$

Similarly, testing for frequencies $\pi/6$, $11\pi/6$ one has to test whether ρ_1 and ρ_2 equal to 0 by using the equation:

$$(1-\sqrt{3}B+B^2)\underline{b_t} = -\rho_1\underline{b_{t-2}} - \rho_2\underline{b_{t-1}} + \sum_{i=1}^{p} \lambda_i(1-\sqrt{3}B+B^2)\underline{b_{t-i}} + \varepsilon_t \tag{46}$$

Similarly, testing for frequencies $2\pi/3$, $4\pi/3$ one has to test whether ρ_1 and ρ_2 equal to 0 by using the equation:

$$(1+B+B^2)\underline{c_t} = -\rho_1\underline{c_{t-2}} - \rho_2\underline{c_{t-1}} + \sum_{i=1}^{p} \lambda_i(1+B+B^2)\underline{c_{t-i}} + \varepsilon_t \tag{47}$$

Similarly, testing for frequencies $\pi/3$, $5\pi/3$ one has to test whether ρ_1 and ρ_2 equal to 0 by using the equation;

$$(1-B+B^2)\underline{d_t} = -\rho_1\underline{b_{t-2}} - \rho_2\underline{d_{t-1}} + \sum_{i=1}^{p} \lambda_i(1-B+B^2)\underline{d_{t-i}} + \varepsilon_t. \tag{48}$$

Notice all the terms in (33) are stationary. Estimation of the system is easily accomplished if the α's known a priori. If they must be estimated, it appears that a generalization of the two-step estimation procedure proposed by Engle & Granger (1987) is available. Namely, estimate the α's using equations (34), (35), (36), (37), (38), (39), (40), respectively, and then estimate the full model using the estimates of the α's. It is conjectured that the least-squares estimates of the remaining parameters would have the same limiting distribution as the estimator knowing the true α's just as in Engle & Granger two-step estimator. Then put the residuals into (42), (43), (44), (45), (46), (47), (48). Then according to the values of ρ_i 's, decide on the result of the test. Then apply the results to equation (41) and find out the error correction coefficients.

CHAPTER 5

5. RESULTS

In this study, I tried to analyze and show the seasonal cointegration and error correction relations of ISE, E, P, M1, M2, A and R for monthly data. First of all, by using HEGY (1990) and Franses (1991), I tried to drive all seasonal cointegration and error correction formulas for monthly data. In shortly, I drived the formulas for $y_{1,t-1}$, $y_{2,t-1}$, $y_{3,t-1}$, $y_{3,t-2}$, $y_{4,t-1}$, $y_{4,t-2}$, $y_{5,t-1}$, $y_{6,t-2}$, $y_{6,t-1}$, $y_{6,t-2}$, $y_{7,t-1}$, $y_{7,t-2}$, $y_{8,t}$, variables that are used to test significance of the frequencies, then by running regression on:

$$\varphi^*(B)y_{8,t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-1} + \pi_4 y_{3,t-2} + \pi_5 y_{4,t-1} + \pi_6 y_{4,t-2} + \pi_7 y_{5,t-1} + \pi_8 y_{6,t-2}$$
$$+ \pi_9 y_{6,t-1} + \pi_{10} y_{6,t-2} + \pi_{11} y_{7,t-1} + \pi_{12} y_{7,t-2} + \mu_t + \varepsilon_t$$

where μ_t can be seasonal dummies, and/or trend, and/or constant, or nothing.

I tried to test the significance of the frequencies. The critical values are taken from Franses (1991). According to this regression results:

ISE, with constant and seasonal dummies and without lag, can not reject frequencies at π_1, π_2, π_4, π_5, π_8, π_{10}, π_{12}, with the values -0.870, -2.170, -3.070, -2.810, 0.005, -2.500 and -0.919, accordingly. Moreover, π_3, and π_6 are significant at 10% level, with values, -2.030 and -3.130, π_7, π_9, and π_{11} are significant at 5% level, with values, -2.310, -3.270 and -2.770. By using the F-test, it shows that only the π_5, and π_6 can not reject the null hypothesis, with the value 4.810.

If I include trend in addition to constant and seasonal dummies, frequencies can not reject null at π_2, π_8, π_{10}, π_{12} , with the values -2.300, -2.050, -2.850, -1.830, accordingly. Moreover, π_3, , π_6 , π_7 are significant at 10% level, with values, -1.840 and -3.060, 0.090, π_1, π_4, π_6, π_9, and π_{11} are significant at 5% level, with values, --3.520, -3.580, -3.430, -3.260, and -1.950. By using the F-test, it shows that all the coefficients reject the null hypothesis.

ISE, with constant and seasonal dummies and with 12 lags, can reject the null hypothesis at frequency at π_7 with the value -1.660, accordingly. Moreover, π_7 are significant at 5% level. By using the F-test, it shows that only the π_3, ... π_{12} can reject the null hypothesis, with the value 5.370.

If I include trend to constant and seasonal dummies, still frequency that can not reject null is π_7 with the value -0.970, accordingly. Moreover, π_7 is significant at 5% level. By using the F-test, it shows that only the π_3, ... π_{12} can reject the null hypothesis, with the value 5.660. You can also see these results from Table 8.

E, with constant and seasonal dummies and without lag, can not reject frequencies at π_1, π_4, π_5, π_6, π_8, π_9, π_{10}, π_{12}, with the values 1.640, 1.490, -2.330, -2.490, 0.890, -

0.290, -0.810, and -0.260, accordingly. Moreover, π_2, π_3, π_7, π_{11}, are significant at 10% level, with values, -2.910, -2.790, -0.820, -3.090. By using the F-test, it shows that π_3 & π_4, π_5 & π_6, π_9 & π_{10} can not reject the null hypothesis, with the values 4.300, 3.010, 0.210.

If I include trend to constant and seasonal dummies, frequencies can not reject null at π_1, π_4, π_5, π_6 , π_8 , π_9 , π_{10} , π_{12} with the values -0.510, 1.390, -2.370, -2.490, -1.090, -0.370, -0.900 and -0.270 accordingly. Moreover, π_2, , π_3 , π_7, π_{11} , are significant at 5% level, with values, -2.860, -2.660, -0.630, -3.050. By using the F-test, it shows that π_3 & π_4, π_5 & π_6, π_9 & π_{10} can not reject the null hypothesis.

E, with constant and seasonal dummies and with 12 lags, can reject the null hypothesis at frequency at π_7 and π_{11} with the values -0.290 and -1.160, accordingly. Moreover, π_7 and π_{11} are significant at 5% level. By using the F-test, it shows that no one of the frequencies can reject the null hypothesis.

If I include trend to constant and seasonal dummies, still frequency that can not reject null is π_7 with the value -0.970. Moreover, π_7 is significant at 5% level. By using the F-test, it shows that only the π_3, ... π_{12} can reject the null hypothesis, with the value 5.660. You can also see these results from Table 14.

M1, with constant and seasonal dummies and without lag, can not reject frequencies at π_1, π_3, π_4, π_5, π_6, π_8, π_9, π_{12}, with the values 0.650, -1.480, -1.970, -1.620,

-2.900, -1.220, -1.980, and -1.420, accordingly. Moreover, π_2, π_7, π_{11}, are significant at 5% level, with values, -3.110, -0.660, -2.870. By using the F-test, it shows that π_3 & π_4, can not reject the null hypothesis, with the value 3.120.

If I include trend to constant and seasonal dummies, frequencies can not reject null at π_1, π_3, π_4, π_5, π_6 , π_8 , π_9 , π_{12} with the values -1.990, -1.430, -2.100, -1.670, -2.920, -1.850, -2.040 and -1.830 accordingly. Moreover, π_2, π_{11}, are significant at 5% level, with values, -3.170 and -2.490. By using the F-test, it shows that π_3 & π_4, can not reject the null hypothesis, with value 3.480.

M1, with constant and seasonal dummies and with 12 lags, can reject the null hypothesis at frequency at π_7 and π_{11} with the values -1.100 and -1.480, accordingly. Moreover, π_7 and π_{11} are significant at 5% level. By using the F-test, it shows that no one of the frequencies can reject the null hypothesis.

If I include trend to constant and seasonal dummies, frequencies that can not reject null are π_7 and π_{11}, with the values -0.430 and -1.190, accordingly. Moreover, they are significant at 5% level. By using the F-test, it shows that only the π_7 & π_8, π_3 ... π_{12} can reject the null hypothesis, with the values 7.670 and 5.490. You can also see these results from Table 11.

M2, with constant and seasonal dummies and without lag, frequencies can not reject null at π_1, π_4, π_5, π_6, π_8, π_{12}, with the values 0.950, -0.960, -2.000, -2.830, 0.650, and 1.180, accordingly. Moreover, π_2, π_3, π_7, π_9, π_{11}, are significant at 5% level, with values, -3.940, -3.030, -1.840, -3.220 and -2.080. By using the F-test, it shows that π_5 & π_6, π_7 & π_8, π_{11} & π_{12}, can not reject the null hypothesis, with the values 4.670, 3.270, 2.330.

If I include trend to constant and seasonal dummies, can not reject frequencies at π_1, π_4, π_5, π_6 , π_8, π_{12} with the values -2.350, -1.220, -2.020, -2.800, 0.110, and 0.770, accordingly. Moreover, π_2, , π_3, , π_9, π_{10} and π_{11}, are significant at 5% level, with values, -3.960, -2.940, -1.350, -3.310, -3.410, -1.730. By using the F-test, it shows that π_5 & π_6, π_7 & π_8, π_{11} & π_{12}, can not reject the null hypothesis, with values 4.500, 3.500, 1.500.

M2, with constant and seasonal dummies and with 12 lags, can reject the null hypothesis at frequency at π_7 and π_{11} with the values -1.100 and -1.480, accordingly. Moreover, π_3 and π_7 are significant at 5% level. By using the F-test, it shows that no one of the frequencies can reject the null hypothesis.

If I include trend to constant and seasonal dummies, frequencies that can not reject null are π_1, π_3 and π_7, with the values -3.030, -1.790 and -1.040, accordingly. You can also see these results from Table 10.

P, with constant and seasonal dummies and without lag, can not reject frequencies at π_1, π_4, π_6, π_7, π_8, π_9, π_{10}, π_{12} with the values 1.160, -0.930, -3.100, 1.060, -2.500, -2.000, -2.100 and -1.430, accordingly. Moreover, π_2, π_3, π_5, π_{11}, are significant at 5%

level, with values, -2.960, -3.320, -3.170, and -2.080. By using the F-test, it shows that π_5 & π_6, π_9 & π_{10}, can not reject the null hypothesis, with the values 4.670, 2.340.

If I include trend to constant and seasonal dummies, can not reject frequencies at π_1, π_4, π_6, π_7, π_8, π_9, π_{10}, π_{12} with the values -0.120,-0.920, -3.060, 1.010, -2.470,-2.000, -2.090 and -1.350 accordingly. Moreover, π_2, π_3, π_5, π_{11} are significant with values, -2.910, -3.250, -3.150, -2.070, accordingly. By using the F-test, it shows that π_5 & π_6, π_9 & π_{10}, can not reject the null hypothesis, with values 4.630 and 2.310.

P, with constant and seasonal dummies and with 12 lags, can reject the null hypothesis at frequency at π_3 and π_{11} with the values -1.860 and -0.800, accordingly. By using the F-test, it shows that π_3 ... π_{12} can reject the null hypothesis.

If I include trend to constant and seasonal dummies, frequencies that can not reject null are π_3 and π_{11}, with the values -1.890, and -1.470, accordingly. By using the F-test, it shows that π_3 ... π_{12} can reject the null hypothesis. You can also see these results from Table 13.

A, with constant and seasonal dummies and without lag, frequencies can not reject null at π_1, π_3, π_4, π_7, π_8, π_9, π_{10}, π_{12} with the values 0.770, -1.600, -3.050, 0.100, -1.340, -2.450, -2.490 and -1.520, accordingly. Moreover, π_2, π_5, π_6, π_{11}, are significant at 5% level, with values, -4.320, -4.150, -3.900, and -1.190, accordingly. By using the F-test, it shows that π_7 & π_8, π_9 & π_{10}, π_{11} & π_{12}, can not reject the null hypothesis, with the values 3.870,4.250, 4.110.

If I include trend to constant and seasonal dummies, can not reject frequencies at π_1, π_3, π_4, π_7, π_8, π_9, π_{10}, π_{12} with the values -1.150,-1.520, -3.110, 0.280, -1.510,-2.400, -2.520 and -1.600 accordingly. Moreover, π_2, π_5, π_6, π_{11} are significant with values, -4.320, -4.140, -3.920, -1.110, accordingly. By using the F-test, it shows that π_3 ... π_{12}, can not reject the null hypothesis, with value 13.120.

A, with constant and seasonal dummies and with 12 lags, can accept the null hypothesis at all frequencies. By using the F-test, it shows that all frequencies can accept the null hypothesis.

If I include trend to constant and seasonal dummies, can accept the null hypothesis at all frequencies. By using the F-test, it shows that all frequencies can accept the null hypothesis.You can also see these results from Table 12.

A, with constant and seasonal dummies and without lag, can not reject frequencies at π_1, π_2, π_3, π_4, π_6, π_8, π_9, π_{10}, π_{12} with the values -1.000, -1.700, -1.700, -1.980, -1.920, -1.180, -2.180, -2.000, and -1.180 accordingly. Moreover, π_6, π_7, π_{11}, are significant with values, -3.600, -0.170, -2.220. By using the F-test, it shows that π_5 & π_6, π_7 & π_8, π_3 ... π_{12}, can reject the null hypothesis, with the values 7.490,5.490, and 14.920.

If I include trend to constant and seasonal dummies, frequencies can not reject null at π_1, π_2, π_3, π_4, π_6, π_8, π_9, π_{10}, π_{12} with the values -0.380, -1.700, -1.620, -2.070, -1.930, -0.080, -1.370, -2.120,-2.020 and -1.330. Moreover, π_6, π_{11} are significant with values, -3.570, -2.030, accordingly. By using the F-test, it shows that π_5 & π_6, π_7 & π_8, π_{11} & π_{12}, $\pi_3 \ldots \pi_{12}$, can reject the null hypothesis, with the values 7.270,5.380, 6.660 and 14.650.

A, with constant and seasonal dummies and with 12 lags, can reject the null hypothesis at frequency at π_7 and π_{11} with the values -0.350 and -1.910, accordingly. By using the F-test, it shows that π_5 & π_6 can reject the null hypothesis.

If I include trend to constant and seasonal dummies, frequencies that can not reject null are π_6, π_7 and π_{11}, with the values -1.030, -0.390, 5.730 accordingly. By using the F-test, it shows that all frequencies can accept the null hypothesis. You can also see these results from Table 9.

After getting these results, to find out the cointegration relations between ISE and P, E, M1, M2, I choose the frequencies which can not reject the null for each variable. For this reason, to find out the cointegration relations between (ISE and E),(ISE and M1), (ISE and M2), (ISE and P) I choose π_1, π_5, π_6, which are common frequencies for variable of our interest for both variable pairs. Then I apply the procedure of seasonal cointegtration that was analyzed at Chapter 2, 3, and 4. I obtain the results that are shown at Tables 15, 16, 17, 18. Results showed that there is a seasonal cointegration between ISE and M1, ISE and M2, ISE and E, ISE and P for frequencies mentioned above.

Also, Table 19, 20, 21, 22 (error correction tables) showed the error correction representation between the cointegrated variables such as ISE and P, E, M1 and finally M2 for the frequencies tested for cointegration. Examinations of tables 19, 20, 21, 22 revealed that almost all coefficients of the ECM's are statistically significant, therefore our choosen frequencies perfectly fit the data.

CHAPTER 6

6. CONCLUSION

The theory of seasonal cointegration of time series is extended to cover series with unit roots at frequencies different from the long run frequency. In particular, seasonal series are studied with a focus upon monthly periodicity.

A proposition on the representation of rational polynomials allows reformulating of time series for monthly case. Based on the least squares fits of transformed variables, similar to well known HEGY regressions, tests for the existence of seasonal as well as zero-frequency unit roots in monthly data are presented.

By extending the definition of cointegration to occur at separate frequencies, the error-correction representation is developed by use of the HEGY (1990). The error correction representation is shown to be a direct generalization of the well-known form, but on properly transformed variables for monthly case.

One reason for the extensive interest in cointegration is its interpretation as testing for the presence (or absence) of a long run equilibrium relationship between a vector of economic time series. The existence of cointegration between a set of economic variables, furthermore, provides a statistical foundation for the use of error correction models, which separate the long run equilibrium relationships between the cointegrated variables from the short run responses. Recently, the questions of testing for cointegration and the estimation of error correction models in the presence of nonstationary seasonality have been addressed.

Notice, that as cointegration at the long-run frequency is interpreted as indication of a 'parallel' long-run movement in the nonstationary series. ISE and E, P, M1, M2 cointegrating at a particular seasonal frequency is interpreted as evidence for a 'parallel' movement in the corresponding seasonal component of the two series which both exhibit a varying seasonal pattern. Moreover, results of the tests showed that for long run frequencies the cointegration relation is positive, but in short run, it is negative. These results are very reasonable for Turkish Economy.

By using the models derived in this thesis, I realize that by applying filters of lags, the number of frequencies that accept the null increases. This shows that using of filters move the unrelated components and by this way model fit the data in much more reasonable sense.

Moreover, by adding trend (addition to constant and seasonal dummies) resulted in, generally, that the situation as the number of frequencies that accept the null hypothesis decreases. This means that trend takes out some part of seasonality which is a general conclusion for this subject.

As a result, I derived the seasonal cointegration and error correction models for monthly time series. Although, Muratoglu-Metin (1994) use the same data set and look for cointegration, they did not deal with these data in the monthly sense. By using these time series models, one can analyze seasonal cointegrations and error correction coefficients for monthly data model.

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APPENDIX A

Table1
Data for Avans, monthly,
86M1-94M5

A

	January	February	March	April	May	June
1986	925	999	1043	1015	977	1048
1987	1163	1208	1208	1220	1213	1221
1988	1655	1719	1785	1699	1634	1799
1989	2164	2333	2173	2154	2241	2749
1990	2019	1525	2052	3096	2845	3157
1991	3128	2779	5080	5367	3972	5504
1992	12190	16282	23018	26152	26238	27143
1993	33150	34752	36943	42412	45693	41165
1994	88465	116846	117773	121444	117138	

	July	August	September	October	November	December
1986	978	999	988	1048	995	1052
1987	1225	1261	1307	1221	1307	1407
1988	1983	1678	1702	1799	1767	2082
1989	2916	2750	3091	2494	2111	2539
1990	3110	3177	2911	3028	2630	2870
1991	4815	4456	6968	11427	10908	13589
1992	24753	23055	24620	25374	26973	31062
1993	36943	37290	45693	41165	61526	70421
1994						

Table 2
Data for Imkb, monthly,
86M1-94M12
ISE

	January	February	March	April	May	June
1986	100	118.87	115.75	112.28	115.13	115.43
1987	216.96	260.76	245.83	269.4	394.79	446.31
1988	857.74	721.23	635.27	553.98	553.07	468.9
1989	379.74	487.09	465.9	533.61	453.05	795.88
1990	3641.25	3516.12	3294.31	3308.23	3852.58	4132.98
1991	4213.48	5102.57	4529.95	3554.25	3626.36	3587.36
1992	4926.19	3664	4076.62	3686.37	3297.36	4407.23
1993	4383.01	5923.61	5864.17	7807.64	8375.75	10778.67
1994	20104.84	15003.59	14087.16	15096.68	14749.1	21752.21

	July	August	September	October	November	December
1986	121.45	138.6	146.67	150.24	160.31	170.86
1987	1012.1	1149.03	1029.25	786.38	890.61	673
1988	492.88	428.06	455.22	404.12	405.84	373.93
1989	801.69	875.98	1475.26	1664.01	1507.54	2217.66
1990	5384.48	4939.23	5085.15	4570.44	3256.96	3255.75
1991	3041.44	3301.29	2937.64	2746.84	4058.47	4369.15
1992	4264.13	4157.83	3976.4	3642.7	3786.24	4004.18
1993	10077.62	12357.02	15079.87	14500.69	18977.16	20682.89
1994	19766.4	25282.43	26825.53	24889.5	28181.04	27257.14

Table 3
Data for Kur, monthly,
86M1-94M12
E

	January	February	March	April	May	June
1986	589.21	597.96	639.65	674.05	678.08	688.96
1987	754.15	762.39	777.65	791.53	807.51	837.8
1988	1082.34	1157.86	1205.59	1252.22	1298.8	1354.04
1989	1853.37	1912.63	1982.71	2059.67	2075.04	2121.12
1990	2334.05	2381.79	2458.83	2502.93	2552.61	2632.95
1991	2996.45	3143.2	3550.5	3801	3985.35	4227.83
1992	5322.59	5684.19	6101.29	6426.63	6718	6889.4
1993	8711.8	9049.7	9380.3	9563.1	9980.65	10484.9
1994	15164.23	17704.92	20586.84	32158.31	33714.26	31682.55

	July	August	September	October	November	December
1986	685.4	686.46	699.71	713.71	749.39	758.79
1987	869.28	892.47	914.63	948.34	960.02	995.96
1988	1421.1	1503.19	1589.62	1725.92	1727.37	1800.04
1989	2145.29	2187.45	2244.5	2281.14	2316.59	2314.5
1990	2669.94	2682.05	2722	2744.62	2777.49	2876.89
1991	4384	4516.55	4654.05	4841.05	4958.14	5059.38
1992	6952.3	7101.7	7280.5	7567.5	8123.2	8360
1993	11186.64	11646.43	11882.32	12508.51	13377.46	14061.7
1994	30969.79	31664	33915.54	34882.19	36258.27	37402.77

Table 4
Data for M1, monthly,
86M1-94M5

M1

	January	February	March	April	May	June
1986	2609.1	2651.3	2851.4	2983.7	3071.6	3304.8
1987	4239.5	4378.7	4521.7	4653.4	5135	5077.2
1988	6711	6454.1	6809.2	6885.8	7122.5	7302.3
1989	9375.2	10037.6	10580	11653.7	11890.7	12760.8
1990	18480.5	18632.4	19520.4	21965.1	21566.8	24653.6
1991	29533.3	29436.3	28570.9	29550.5	30089.8	33419.4
1992	39672.5	41904.2	43716.9	46127.4	46806.3	50956.5
1993	71080.2	74830.6	86118.9	81397	94228.4	90423.6
1994	109176.3	112820.8	110583.7	127604	140616.4	153093.6

	July	August	September	October	November	December
1986	3407	3612	3784.7	3725.4	3983.2	4361.8
1987	5638.8	5759	5993.9	6377.6	6232.6	8268.2
1988	8220.6	8326.2	8754	8541.7	8697.6	11243.7
1989	14161.6	14966.5	16310.8	17097.8	17224.6	20358.1
1990	24680.3	26239.8	26843	26818.4	26390.4	29326.4
1991	33446	29550.5	39429.8	40501.9	39038.5	42115.9
1992	53932.5	46127.4	60087	64226.4	65634	70520.6
1993	97982.3	81397	105198.5	112558.8	109577.5	132307.8
1994	179751.3	127604	205486.9	209758.5	210054.6	238981

Table 5
Data for M2, monthly,
86M1-94M12

M2

	January	February	March	April	May	June
1986	7776	8025.2	8427.56	8576.79	8874.66	9128.49
1987	10978.79	11234.49	11535.71	11757.63	12237.23	12319.33
1988	15109.63	15374.46	15684.24	16189.07	16466.07	16805.59
1989	26824.53	28818.97	30340.2	32043.12	32737.1	34065.1
1990	48794	49851.87	51657.23	54379.36	55403.17	59332.33
1991	71363.86	73760.43	75113.93	76805.16	80433.8	85795.02
1992	117627.8	123689.8	129827.4	134643	139046.6	143203.4
1993	192769.5	203702.6	212445.3	215142.5	227685.7	223933.6
1994	276490.2	289837.8	280607.2	332736.7	404103.2	475032.8

	July	August	September	October	November	December
1986	9227.23	9557.37	9828.76	10109.9	10481.37	11154.21
1987	13023.6	13430.44	13949.27	14378.66	14334.94	16309.37
1988	18164.81	18858.62	19665.5	21012.6	22501.57	26970.07
1989	35806.07	37452.93	39804.91	42784.31	44704.47	48823.67
1990	60271.98	62643.5	64135.86	66084.76	67442.1	70731.43
1991	88577.62	93824.7	100228.8	104284.6	107366.6	114807
1992	150093.1	157514.8	162375.6	169263.3	175643.1	194572.5
1993	234723	241474.2	247617.2	256474.2	259694.1	291974.7
1994	557166.9	550846.1	563013	558715.1	586029.9	636652.6

Table 6
Data for Price, monthly,
86M1-94M12
P

	January	February	March	April	May	June
1986	1409.5	1434	1452.7	1458.2	1485.9	1521.4
1987	1837.2	1886.8	1957.3	1998.3	2096.9	2094.7
1988	2932.8	3060.3	3324.1	3486.6	3568.8	3646
1989	5112.8	5334.4	5469.2	5666.5	5809.4	6113.7
1990	8492.7	8851.8	9285.4	9656.4	9828.6	10126.1
1991	13697.2	14397.8	14903.4	15439.8	16050.3	16858.4
1992	24484.5	25773.1	26739.9	27087.2	27344.6	28146.4
1993	39859	41223.2	42554.2	43585.2	45626.4	47886.3
1994	66960.6	70957.9	74563.3	94060.4	102017.4	105781.4

	July	August	September	October	November	December
1986	1549.9	1563.4	1599.4	1716.8	1755.9	1784.7
1987	2134.8	2171.4	2234.9	2342.2	2488.5	2767.2
1988	3746.9	3874.6	4062.1	4365.3	4665.7	4848.3
1989	6460.8	6716.6	7033	7566.8	7903.7	8182.5
1990	10315.9	10608.5	11302	12118	12839.2	13140.9
1991	17326.1	18110.9	18999.8	20244.4	21527.4	22484.4
1992	29088.4	30239.6	32393.4	35056.9	36529.3	37748.4
1993	51242.5	52571.5	54698.3	58242.8	62524.6	64695.4
1994	109424.6	111796.4	118202.8	129547.5	139730.9	150181.2

Table 7
Data for Rsec, monthly,
86M1-94M5
R

	January	February	March	April	May	June
1986	54.15	53.38	53.98	52.81	52.63	51.73
1987	51.02	43.29	44	46.7	47.25	47.5
1988	47.04	61.09	63.31	64.73	66	62.59
1989	67.45	64.33	53.93	50.59	54.59	59.04
1990	50.07	50.25	50.41	50.54	50.33	50.38
1991	60.08	65.44	69.7	72.88	75.05	60.99
1992	71.94	71.51	71.48	72.5	74.41	77.38
1993	78.08	79.98	82.26	83.87	85.2	85.88
1994	94	125	129.99	126.58	222.54	

	July	August	September	October	November	December
1986	51.52	51.79	52.07	51.661	49.34	46.34
1987	41.5	42.2	47.93	52.07	51.91	49.58
1988	60.82	59.04	58.36	72.11	70.23	67.65
1989	59.86	65.91	65.63	57.43	51.73	50.77
1990	50.4	50.47	50.63	52.31	53.7	58.88
1991	61	65.83	70.66	75.12	76.98	72.98
1992	78.16	77.62	77.23	77.51	77.63	77.83
1993	86.51	87.37	87.97	86.69	87.91	89.23
1994						

Table 8
Seasonal frequency calculation results for ISE

ISE *(1/86 -12/95)*

	without lag		*with 12 lags*	
	C+S	C+S+T	C+S	C+S+T
$t{:}\pi_1$	-0.870	-3.520 **	-0.030	-1.910
$t{:}\pi_2$	-2.170	-2.300	-1.900	-1.980
$t{:}\pi_3$	-2.030 *	-1.840 *	-0.830	-0.800
$t{:}\pi_4$	-3.070	-3.580 **	-2.090	-2.210
$t{:}\pi_5$	-2.810	-3.060 *	-2.500	-2.530
$t{:}\pi_6$	-3.130 *	-3.430 **	-2.310	-2.400
$t{:}\pi_7$	-2.310 **	0.090 *	-1.660 **	-0.970 **
$t{:}\pi_8$	0.005	-2.050	0.280	-0.360
$t{:}\pi_9$	-3.270 **	-3.260 **	-2.420	-2.440
$t{:}\pi_{10}$	-2.500	-2.850	-2.450	-2.560
$t{:}\pi_{11}$	-2.770 **	-1.950 **	-0.120	0.030
$t{:}\pi_{12}$	-0.919	-1.830	-1.790	-2.030
$F{:}\pi_3$ & π_4	7.400 **	8.850 **	2.610	2.840
$F{:}\pi_5$ & π_6	4.810	5.900 **	3.210	3.380
$F{:}\pi_7$ & π_8	10.460 **	11.690 **	3.700	3.380
$F{:}\pi_9$ & π_{10}	6.090 **	6.650 **	4.000	4.230
$F{:}\pi_{11}$ & π_{12}	7.860 **	8.270 **	2.440	2.860
$F{:}\pi_3 \ldots \pi_{12}$	43.060 **	45.910 **	5.370 **	5.660 **

* Significant at 10% level

** Significant at 5% level

C+S, auxiliary regression contains constant and 12 seasonal dummies.

C+S+T, auxiliary regression contains constant, 12 seasonal dummies and trend.

Table 9
Seasonal frequency calculation results for
R

R *(1/86-5/95)*

	without lag		*with 12 lags*	
	C+S	C+S+T	C+S	C+S+T
$t{:}\pi_1$	-1	-0.380	1.640	0.990
$t{:}\pi_2$	-1.700	-1.700	-0.780	-0.770
$t{:}\pi_3$	-1.700	-1.620	-0.620	-0.620
$t{:}\pi_4$	-1.980	-2.070	-1.490	-1.460
$t{:}\pi_5$	-3.600 **	-3.570 **	-2.770	-2.750
$t{:}\pi_6$	-1.920	-1.930	-1.030	-1.030 *
$t{:}\pi_7$	-0.170 *	0.080	-0.350 **	-0.390 **
$t{:}\pi_8$	-1.180	-1.370	-0.420	-0.370
$t{:}\pi_9$	-2.180	-2.120	-0.950	-0.950
$t{:}\pi_{10}$	-2.000	-2.020	-1.020	-1.000
$t{:}\pi_{11}$	-2.220 **	-2.030 **	-1.910 **	-0.18
$t{:}\pi_{12}$	-1.180	-1.330	0.120	1.310
$F{:}\pi_3$ & π_4	3.660	3.720	1.390	5.730 **
$F{:}\pi_5$ & π_6	7.490 **	7.270 **	5.800 **	1.470
$F{:}\pi_7$ & π_8	5.490 *	5.380 *	1.500	0.650
$F{:}\pi_9$ & π_{10}	3.160	3.110	0.670	2.290
$F{:}\pi_{11}$ & π_{12}	3.720	6.660 **	2.340	3.730
$F{:}\pi_3 \ldots \pi_{12}$	14.920 **	14.650 **	3.820	1.770

* Significant at 10% level

** Significant at 5% level

C+S, auxiliary regression contains constant and 12 seasonal dummies.

C+S+T, auxiliary regression contains constant, 12 seasonal dummies and trend.

Table 10
Seasonal frequency calculation results for
M2

_M2_ _(1/86-12/95)_

	without lag		*with 12 lags*	
	C+S	C+S+T	C+S	C+S+T
$t{:}\pi_1$	0.950	-2.350	0.800	-3.030 *
$t{:}\pi_2$	-3.940 **	-3.960 **	-1.400	-1.370
$t{:}\pi_3$	-3.030 **	-2.940 **	-1.850 *	-1.790 *
$t{:}\pi_4$	-0.960	-1.220	-0.570	-0.860
$t{:}\pi_5$	-2.000	-2.020	-1.440	-1.510
$t{:}\pi_6$	-2.830	-2.800	-1.960	-2.050
$t{:}\pi_7$	-1.840 **	-1.350 **	-1.650 **	-1.040 **
$t{:}\pi_8$	0.650	0.110	1.080	0.330
$t{:}\pi_9$	-3.220 **	-3.310 **	-0.380	-0.450
$t{:}\pi_{10}$	-3.160 *	-3.410 **	-0.860	-1.090
$t{:}\pi_{11}$	-2.080 **	-1.730 **	0.610	-0.180
$t{:}\pi_{12}$	1.180	0.770	0.090	-0.300
$F{:}\pi_3$ & π_4	5.140 *	5.500 *	1.870	2.210
$F{:}\pi_5$ & π_6	4.670	4.500	1.870	2.210
$F{:}\pi_7$ & π_8	3.270	3.500	1.500	1.400
$F{:}\pi_9$ & π_{10}	7.010 **	8.000 **	0.370	0.460
$F{:}\pi_{11}$ & π_{12}	2.330	1.500	0.000	0.000
$F{:}\pi_3 \ldots \pi_{12}$	28.400 **	29.900 **	1.650	1.770

* Significant at 10% level

** Significant at 5% level

C+S, auxiliary regression contains constant and 12 seasonal dummies.

C+S+T, auxiliary regression contains constant, 12 seasonal dummies and trend.

Table 11
Seasonal frequency calculation results for
M1

M1 ***(1/86-12/95)***

	without lag		*with 12 lags*	
	C+S	C+S+T	C+S	C+S+T
$t:\pi_1$	0.650	-1.990	1.330	-3.230 *
$t:\pi_2$	-3.110 **	-3.170 **	-1.390	-1.500
$t:\pi_3$	-1.480	-1.430	-1.370	-1.320
$t:\pi_4$	-1.970	-2.100	-1.410	-1.830
$t:\pi_5$	-1.620	-1.670	-0.480	-0.630
$t:\pi_6$	-2.900	-2.920	-1.560	-1.710
$t:\pi_7$	-0.660 **	-0.010 *	-1.100 **	-0.430 **
$t:\pi_8$	-1.220	-1.850	0.130	-1.250
$t:\pi_9$	-1.980	-2.040	-1.450	-1.530
$t:\pi_{10}$	-3.170 *	-3.290 *	-1.270	-1.580
$t:\pi_{11}$	-2.870 **	-2.490 **	-1.480 **	-1.190 **
$t:\pi_{12}$	-1.420	-1.830	-0.490	-1.080
$F:\pi_3$ & π_4	3.120	3.480	1.930	2.550
$F:\pi_5$ & π_6	6.860 **	6.750 **	3.210	3.250
$F:\pi_7$ & π_8	10.610 **	12.190 **	4.060	7.670 **
$F:\pi_9$ & π_{10}	5.200 *	5.660 *	1.160	1.620
$F:\pi_{11}$ & π_{12}	11.440 **	11.970 **	2.120	2.790
$F:\pi_3 \ldots \pi_{12}$	30.580 **	32.360 **	3.980	5.490 **

* Significant at 10% level

** Significant at 5% level

C+S, auxiliary regression contains constant and 12 seasonal dummies.

C+S+T, auxiliary regression contains constant, 12 seasonal dummies and trend.

Table 12
Seasonal frequency calculation results for
A

A *(1/86-5/95)*

	without lag		*with 12 lags*	
	C+S	C+S+T	C+S	C+S+T
t:π_1	0.770	-1.150	0.870	-0.730
t:π_2	-4.320 **	-4.320 **	-1.120	-1.100
t:π_3	-1.600	-1.520	-1.610	-1.540
t:π_4	-3.050	-3.110	-2.270	-2.320
t:π_5	-4.150 **	-4.140 **	-2.580	-2.590
t:π_6	-3.900 **	-3.920 **	-2.160	-2.190
t:π_7	0.100	0.280	0.370	0.450
t:π_8	-1.340	-1.510	-1.000	-1.080
t:π_9	-2.450	-2.400	-1.440	-1.370
t:π_{10}	-2.490	-2.520	-1.310	-1.310
t:π_{11}	-1.190 **	-1.110 **	-0.360	-0.310
t:π_{12}	-1.520	-1.600	-1.060	-1.090
F:π_3 & π_4	6.380 **	6.430 **	4.290	4.290
F:π_5 & π_6	9.020 **	9.000 **	3.320	3.360
F:π_7 & π_8	3.870	3.990	1.070	1.160
F:π_9 & π_{10}	4.250	4.210	1.280	1.230
F:π_{11} & π_{12}	4.110	4.110	1.140	1.120
F:π_3 ... π_{12}	13.240 **	13.120 **	3.060	3.060

* Significant at 10% level

** Significant at 5% level

C+S, auxiliary regression contains constant and 12 seasonal dummies.

C+S+T, auxiliary regression contains constant, 12 seasonal dummies and trend.

Table 13
Seasonal frequency calculation results for
P

P *(1/86-12/95)*

	without lag		*with 12 lags*	
	C+S	C+S+T	C+S	C+S+T
t:π_1	1.160	-0.120	1.590	0.950
t:π_2	-2.960 **	-2.910 **	-1.100	-1.110
t:π_3	-3.320 **	-3.250 **	-1.860 **	-1.890 **
t:π_4	-0.930	-0.920	-0.910	-0.930
t:π_5	-3.170 **	-3.150 *	-1.440	-1.360
t:π_6	-3.100	-3.060	0.610	-1.460
t:π_7	1.060	1.010	-1.750	0.750
t:π_8	-2.500	-2.470	-1.290	-1.740
t:π_9	-2.000	-2.000	-0.560	-1.210
t:π_{10}	-2.100	-2.090	-1.540	-0.480
t:π_{11}	-2.080 **	-2.070 **	-0.800 *	-1.470 **
t:π_{12}	-1.430	-1.350	-0.720	-0.900
F:π_3 & π_4	6.260 **	6.170 **	2.180	2.100
F:π_5 & π_6	4.690	4.630	1.450	0.700
F:π_7 & π_8	7.820 **	7.710 **	1.450	2.800
F:π_9 & π_{10}	2.340	2.310	0.720	0.720
F:π_{11} & π_{12}	6.260 **	6.170 **	2.900	2.100
F:π_3 ... π_{12}	23.790 **	23.300 **	14.540 **	12.100 **

* Significant at 10% level

** Significant at 5% level

C+S, auxiliary regression contains constant and 12 seasonal dummies.

C+S+T, auxiliary regression contains constant, 12 seasonal dummies and trend.

Table 14
Seasonal frequency calculation for
E

E *(1/86-12/95)*

	without lag		*with 12 lags*	
	C+S	C+S+T	C+S	C+S+T
$t{:}\pi_1$	1.640	-0.510	1.860	0.980
$t{:}\pi_2$	-2.910 **	-2.860 **	-1.640	-1.590
$t{:}\pi_3$	-2.790 **	-2.660 **	-1.360	-1.410
$t{:}\pi_4$	1.490	1.390	0.480	0.510
$t{:}\pi_5$	-2.330	-2.370	-0.270	-0.200
$t{:}\pi_6$	-2.490	-2.490	-0.420	-0.370
$t{:}\pi_7$	-0.820 **	-0.630 **	-0.290 **	-0.500 **
$t{:}\pi_8$	-0.890	-1.090	-0.130	0.210
$t{:}\pi_9$	-0.290	-0.370	-0.070	0.050
$t{:}\pi_{10}$	-0.810	-0.900	-1.590	-1.350
$t{:}\pi_{11}$	-3.090 **	-3.050 **	-1.160 **	-1.270 **
$t{:}\pi_{12}$	-0.260	-0.270	-0.740	-0.800
$F{:}\pi_3$ & π_4	4.300	4.130	1.080	1.070
$F{:}\pi_5$ & π_6	3.010	3.480	0.180	0.170
$F{:}\pi_7$ & π_8	5.370 *	5.660 *	0.360	0.170
$F{:}\pi_9$ & π_{10}	0.210	0.430	1.620	1.250
$F{:}\pi_{11}$ & π_{12}	6.450 **	6.530 **	1.620	1.970
$F{:}\pi_3 \ldots \pi_{12}$	23.700 **	22.600 **	1.510	1.500

* Significant at 10% level

** Significant at 5% level

C+S, auxiliary regression contains constant and 12 seasonal dummies.

C+S+T, auxiliary regression contains constant, 12 seasonal dummies and trend.

Table 15 Results of Cointegration Tests between ISE and E

	α_1	ρ	α_6	α_7	π_5	π_6	$F:\pi_5\&\pi_6$
value	1.26	-0.0062	-0.7452	-1.148	-0.6847	-0.7624	9.625
SE	0.05	0.0105	0.5006	0.5072	0.1775	0.1796	
t	24.79	-0.5937	-1.488	-2.2636	-3.85	-4.244	
R^2	0.88	0.941	0.226	0.226	0.936	0.936	

* $F:\pi_5\&\pi_6$ can reject the null at 0.50 & 0.90 but can not reject at 0.95, 0.975 & 0.99

* $t:\pi_5$ can reject the null at 0.10 but can not reject at 0.01, 0.025 & 0.05

* $t:\pi_6$ can reject the null at 0.01, 0.025, 0.05, 0.95, 0.975, 0.99

Table 16 Results of Cointegration Tests between ISE and M1

	α_1	ρ	α_6	α_7	π_5	π_6	$F:\pi_5\&\pi_6$
value	1.1809	-0.019	-0.6275	-0.2721	-0.607	-0.6697	6.877
SE	0.0336	0.0161	0.4988	0.499	0.179	0.1802	
t	35.17	-1.1806	-1.258	-0.5454	-3.3908	-3.7162	
R^2	0.937	0.909	0.194	0.194	0.943	0.943	

* $F:\pi_5\&\pi_6$ can reject the null at 0.50 but can not reject at 0.90, 0.95, 0.975 & 0.99

* $t:\pi_5$ can reject the null at 0.01, 0.025, 0.05, 0.1

* $t:\pi_6$ can reject the null at 0.01, 0.025, 0.05, 0.95, 0.975, 0.99

Table 17 Results of Cointegration Tests between ISE and M2

	α_1	ρ	α_6	α_7	π_5	π_6	$F:\pi_5\&\pi_6$
value	1.1479	-0.0204	-1.4886	-0.9404	-0.7327	-0.812	8.357
SE	0.0367	0.0159	0.6654	0.6645	0.1951	0.1952	
t	31.25	-1.2847	-2.2371	-1.4153	-3.7551	-4.1481	
R^2	0.921	0.932	0.214	0.214	0.943	0.943	

* $F:\pi_5\&\pi_6$ can reject the null at 0.50 & 0.90 but can not reject at 0.95, 0.975 & 0.99

* $t:\pi_5$ can reject the null at 0.01but can not reject at 0.01, 0.025, 0.05

* $t:\pi_6$ can reject the null at 0.01, 0.025, 0.05, 0.95, 0.975, 0.99

Table 18 Results of Cointegration Tests between ISE and P

	α_1	ρ	α_6	α_7	π_5	π_6	F:π_5&π_6
value	1.0696	-0.0157	-0.9091	-2.879	-0.7776	-0.8806	10.22
SE	0.0356	0.0138	0.9208	0.9003	0.1943	0.1978	
t	30.077	-1.1401	-0.9873	-3.197	-4.0015	-4.4525	
R^2	0.916	0.933	0.259	0.259	0.937	0.937	

* F:π_5&π_6 can reject the null at 0.50, 0.90 & 0.95 but can not reject at 0.975 & 0.99

* t:π_5 can reject the null at 0.01but can not reject at 0.01, 0.025, 0.05

* t:π_6 can reject the null at 0.01, 0.025, 0.05, 0.95, 0.975, 0.99

Table 19 Results of Error Correction Model between ISE and E

	β	γ_1	γ_2	γ_3
value	-1.76	-0.03	-1.316	-1.198
SE	0.2059	0.0086	0.1177	0.1182
t	-8.54	-3.55	-11.183	-10.144
R^2	0.701			

Table 20 Results of Error Correction Models between ISE and M1

	β	γ_1	γ_2	γ_3
value	1.598	-0.0239	-1.1423	-1.0705
SE	0.528	0.0124	0.1171	0.1132
t	-3.02	-1.9305	-9.7574	-9.4047
R^2	0.75			

Table 21 Results of Error Correction Models between ISE and M2

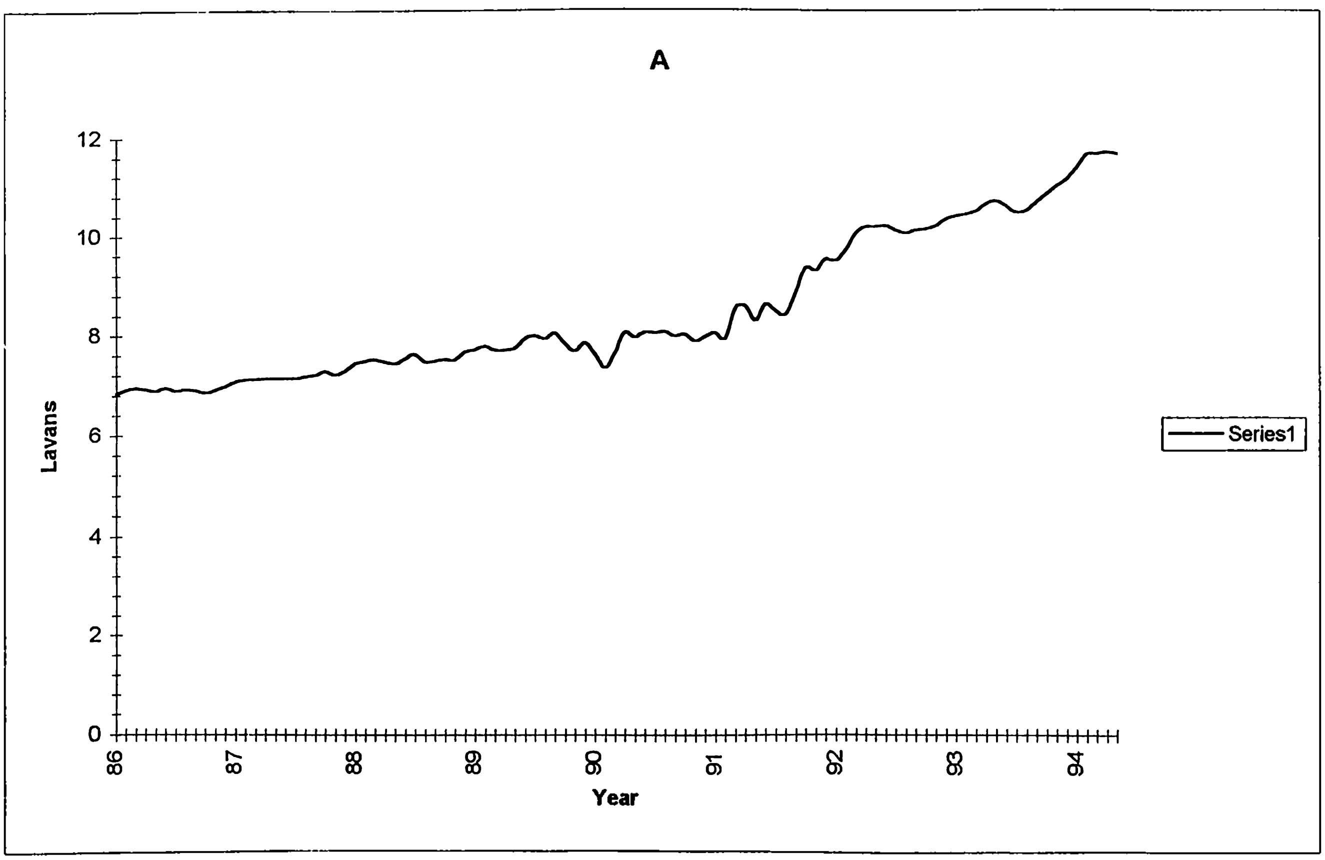
	β	γ_1	γ_2	γ_3
value	-1.5235	-0.043	-1.3723	-1.263
SE	0.3647	0.0111	0.1093	0.1084
t	-4.177	-3.8906	-12.549	-11.645
R^2	0.7			

Table 22 Results of Error Correction Models between ISE and P

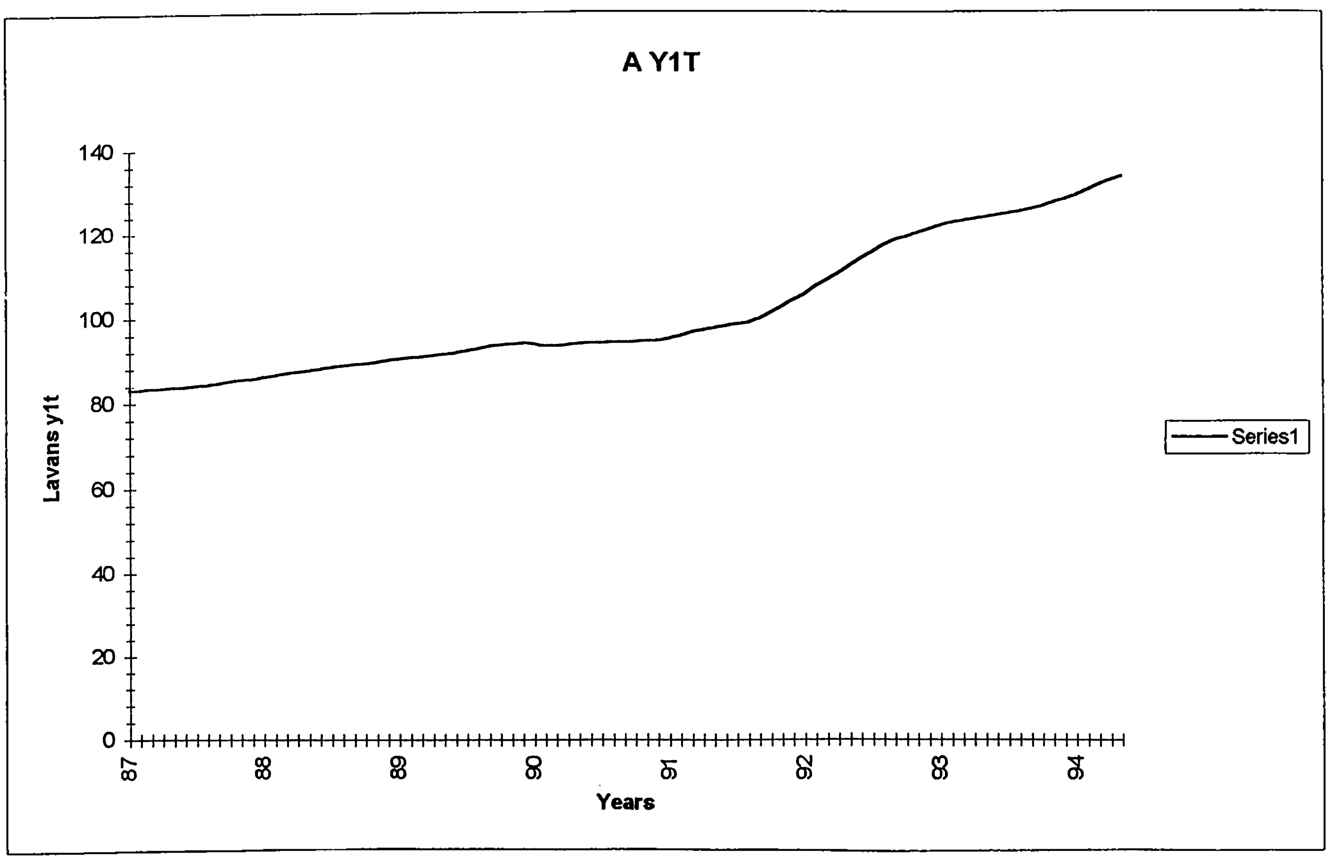
	β	γ_1	γ_2	γ_3
value	-3.2984	-0.0361	-1.3939	-1.2871
SE	0.4477	0.0102	0.1179	0.1194
t	-7.3663	-3.555	-11.825	-10.776
R^2	0.699			

APPENDIX B

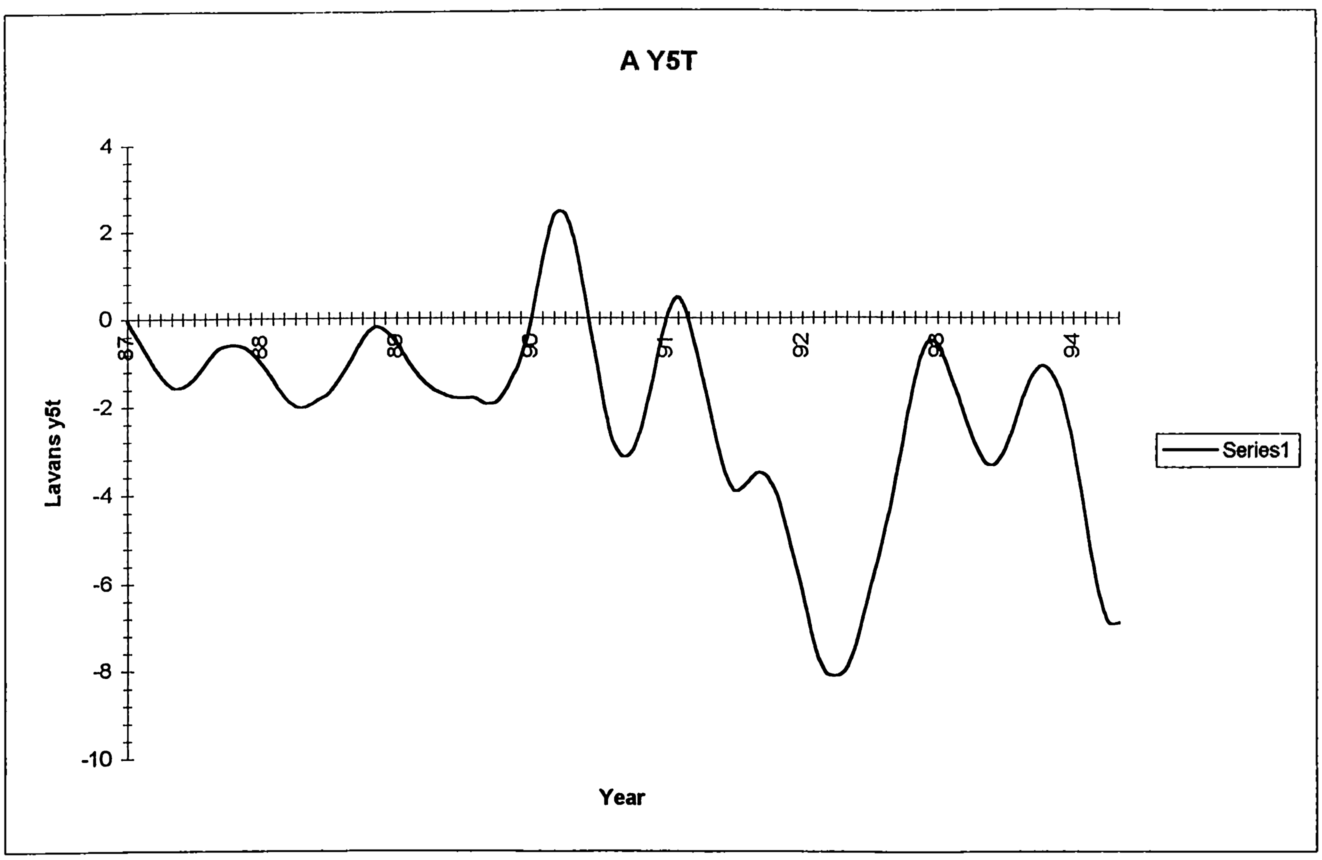
Graph 1
A for years between
86M1-94M5



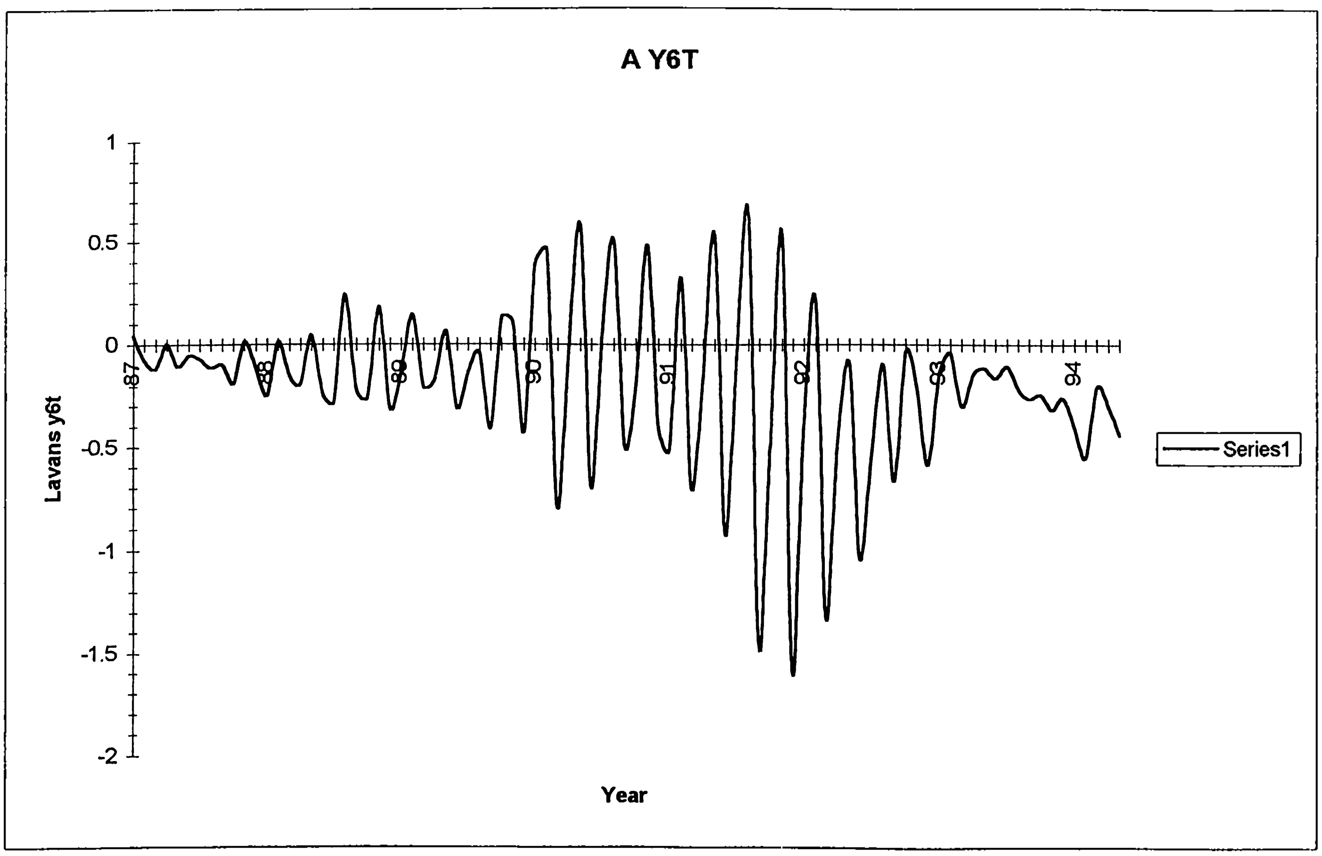
Graph 2
A y1t for years between
87M1-94M5



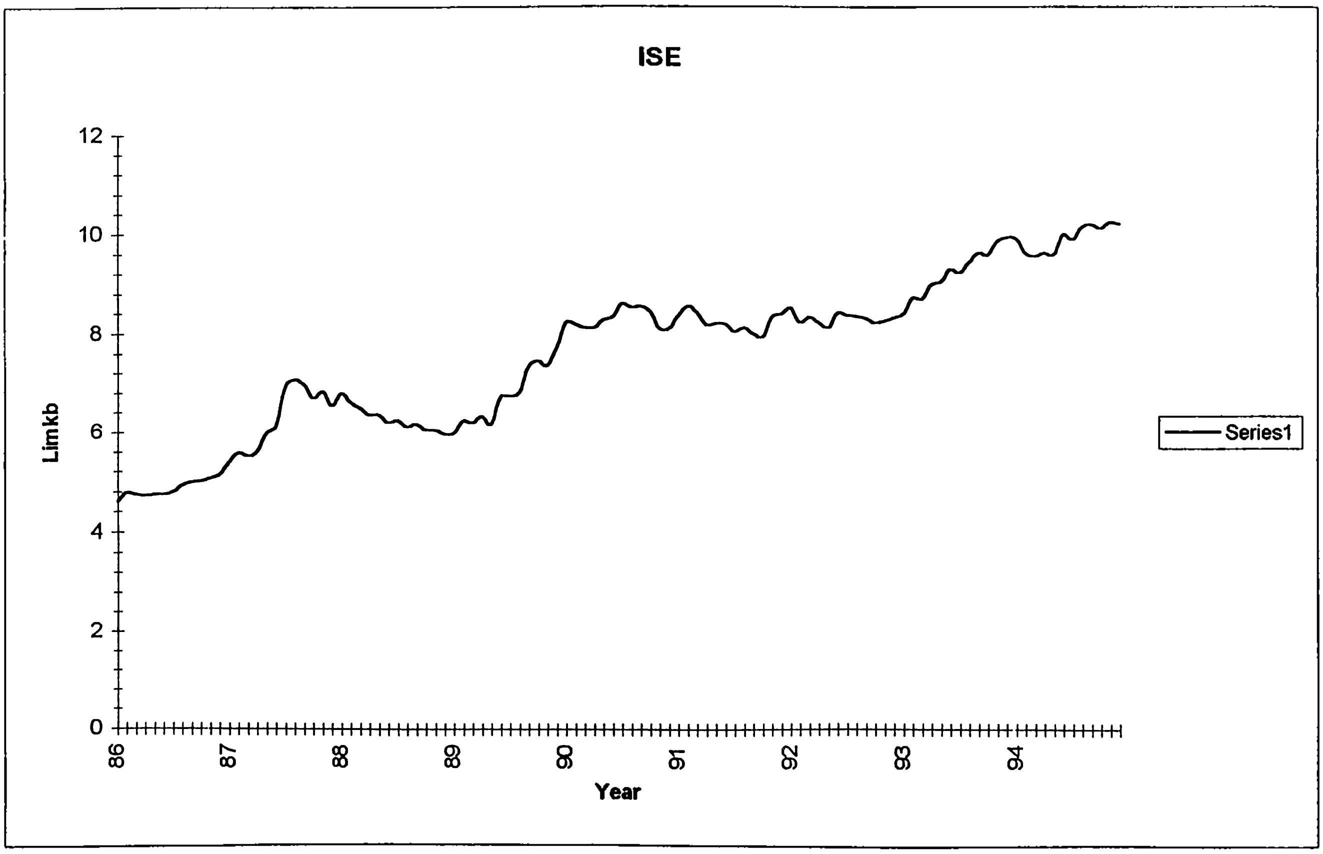
Graph 3
A y5t for years between
87M1-94M5



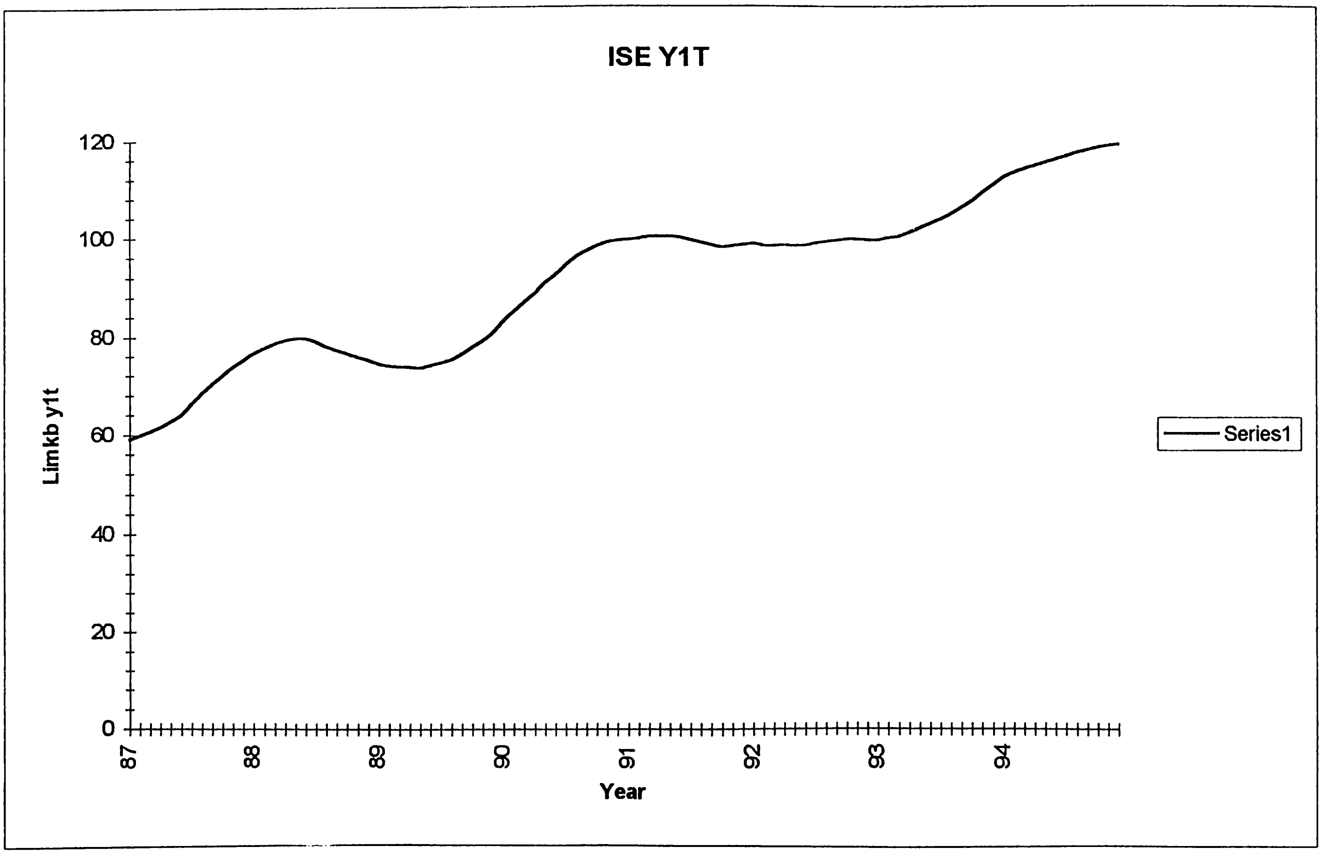
Graph 4
A y6t for years between
87M1-94M5



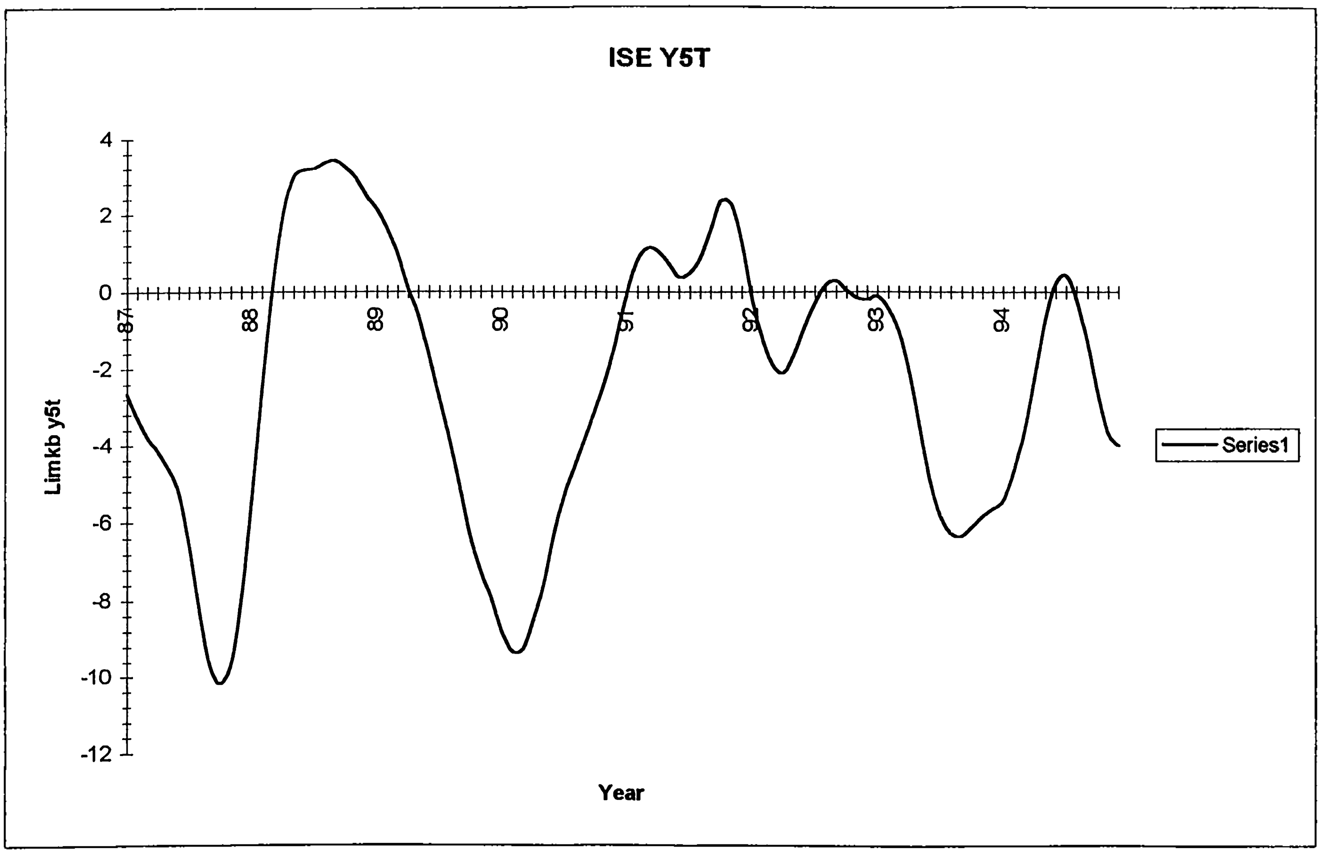
Graph 5
ISE for years between
86M1-94M12



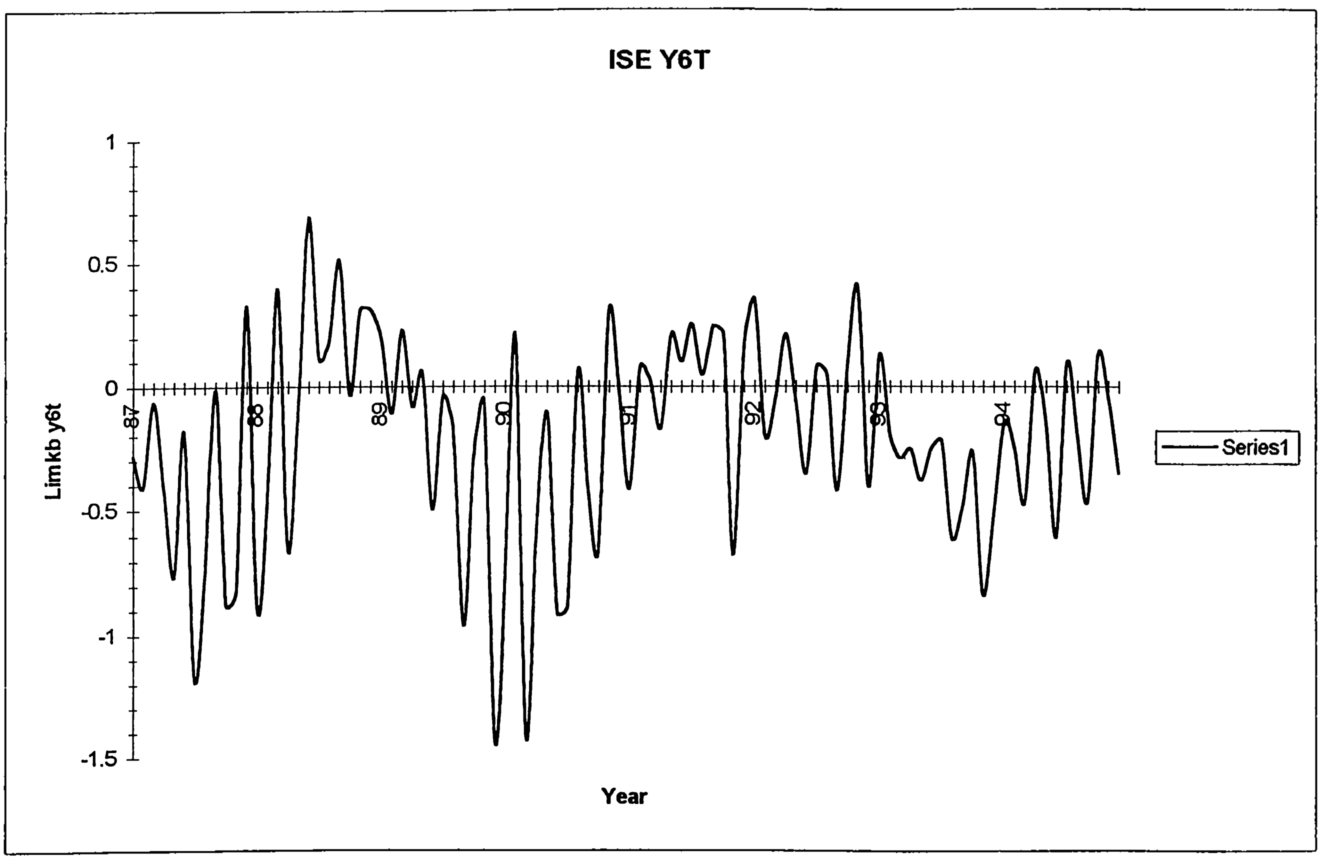
Graph 6
ISE y1t for years between
87M1-94M12



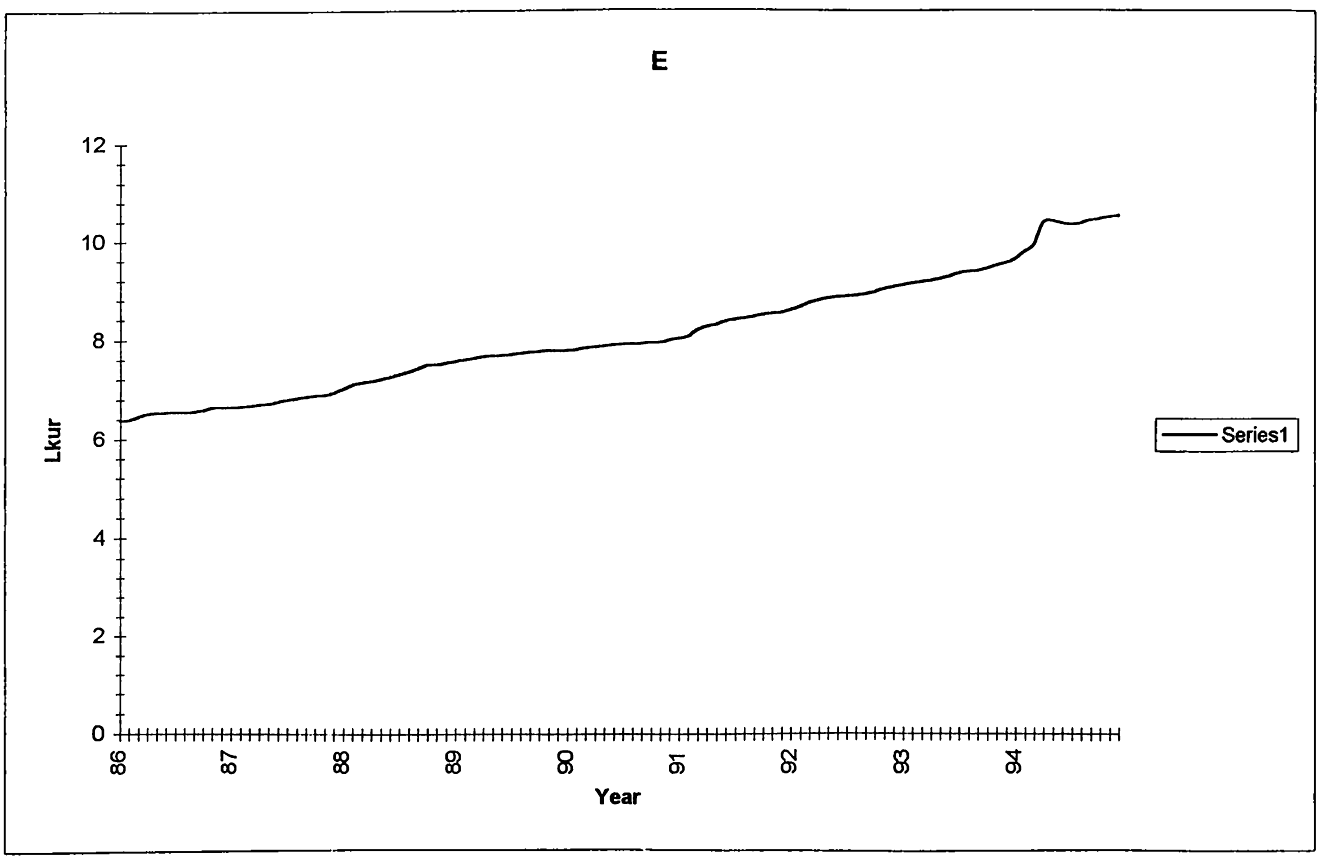
Graph 7
ISE y5t for years between
87M1-94M12



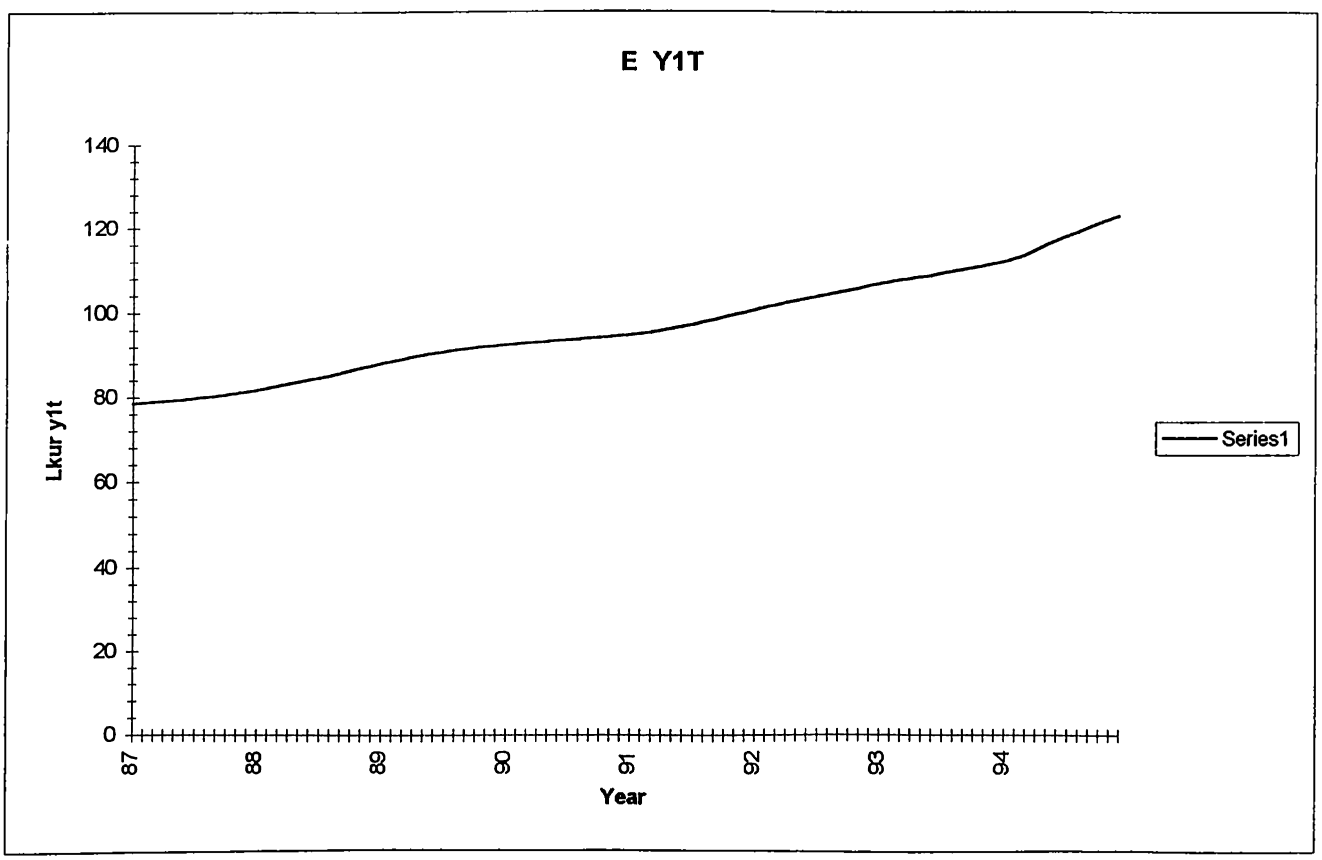
Graph 8
ISE y6t for years between
87M1-94M12



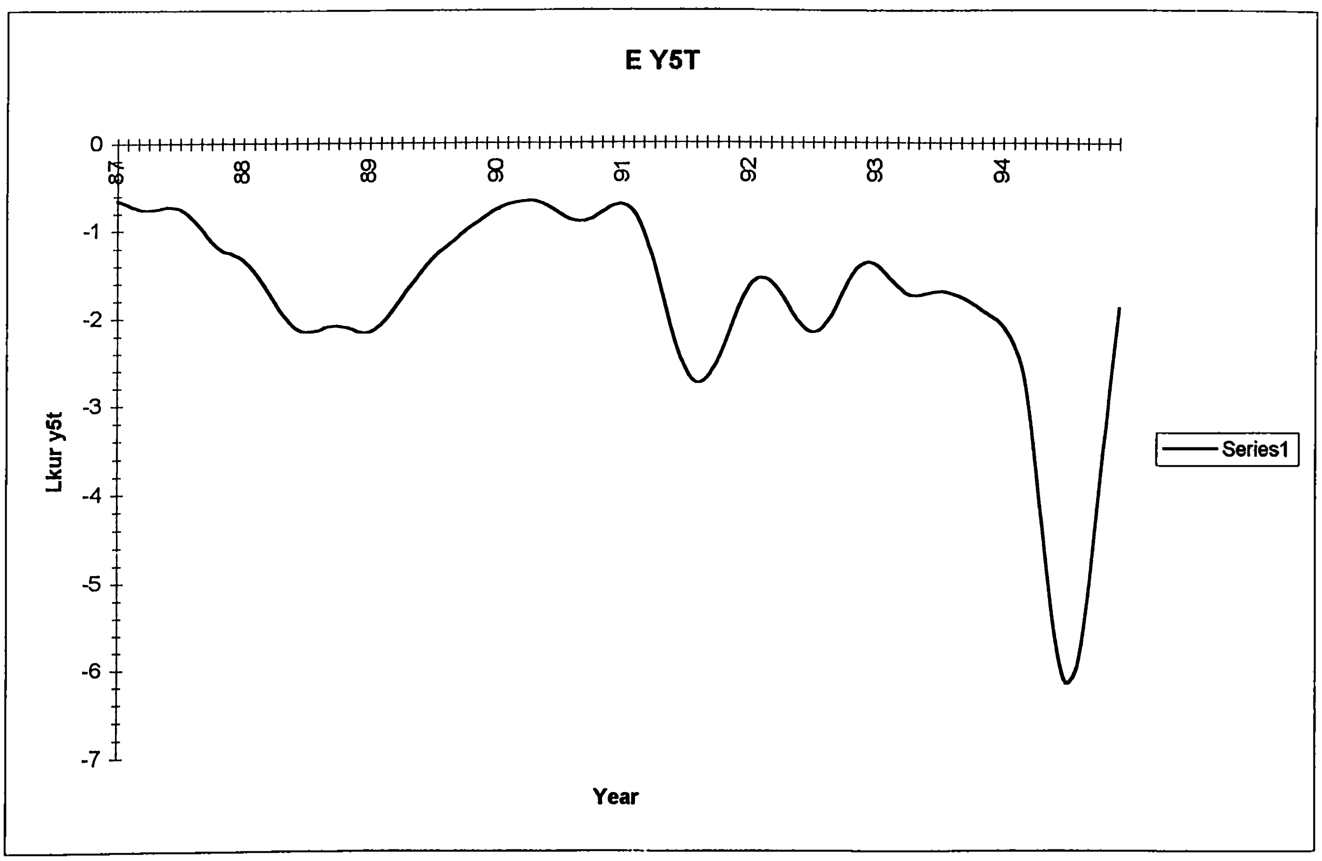
Graph 9
E for years between
86M1-94M12



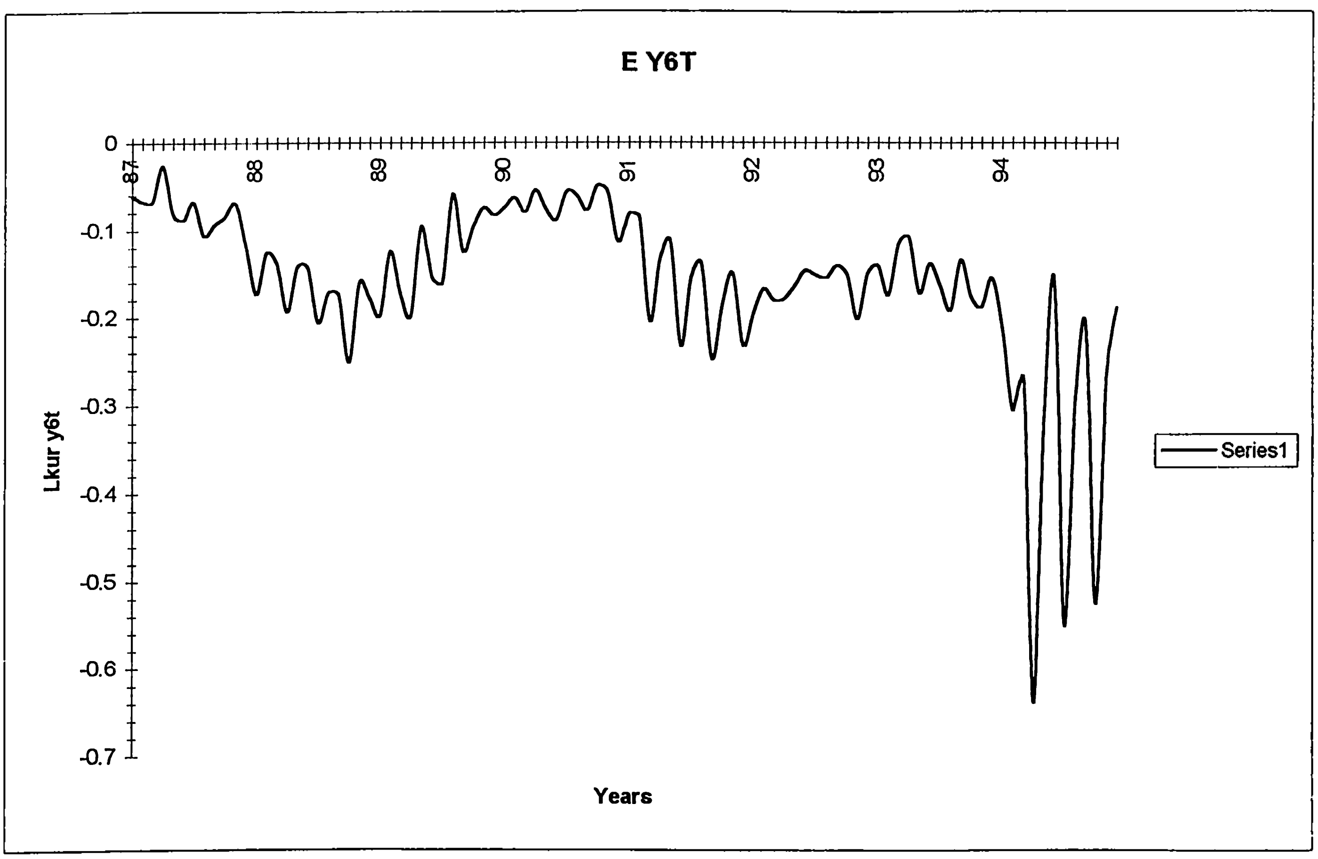
Graph 10
E y1t for years between
87M1-94M12



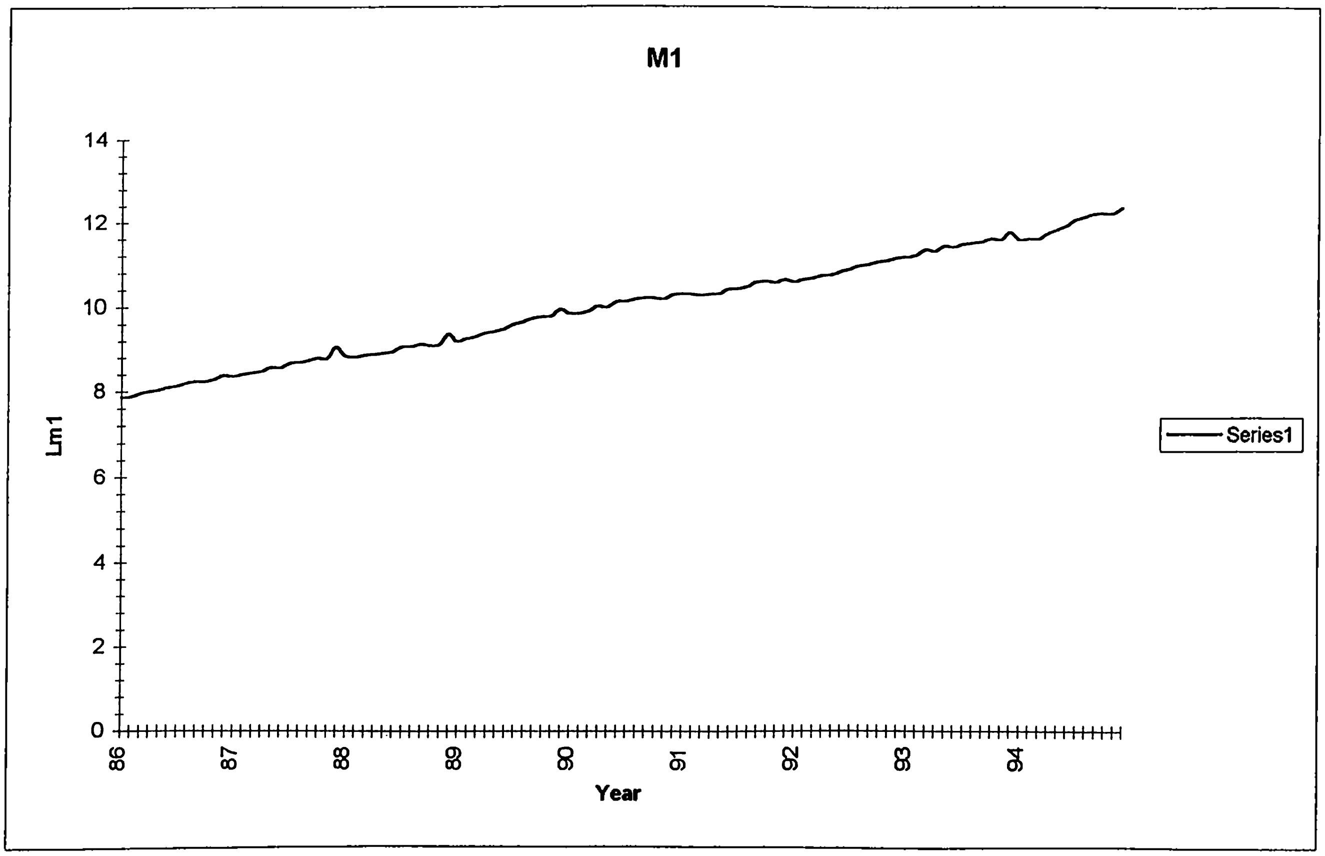
Graph 11
E y5t for years between
87M1-94M12



Graph 12
E y6t for years between
87M1-94M12



Graph 13
M1for years between
86M1-94M12



Graph 14
M1 y1t for years between
87M1-94M12

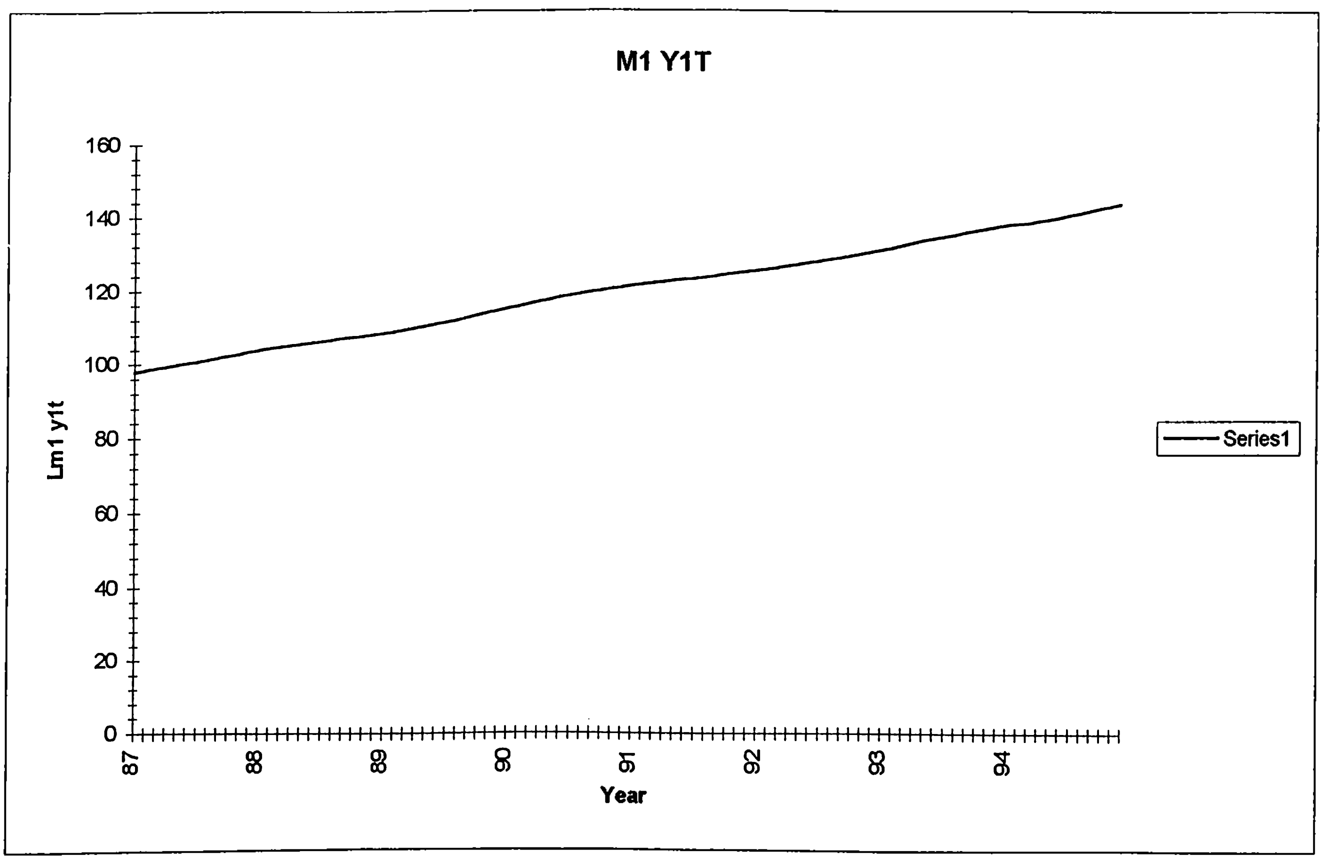
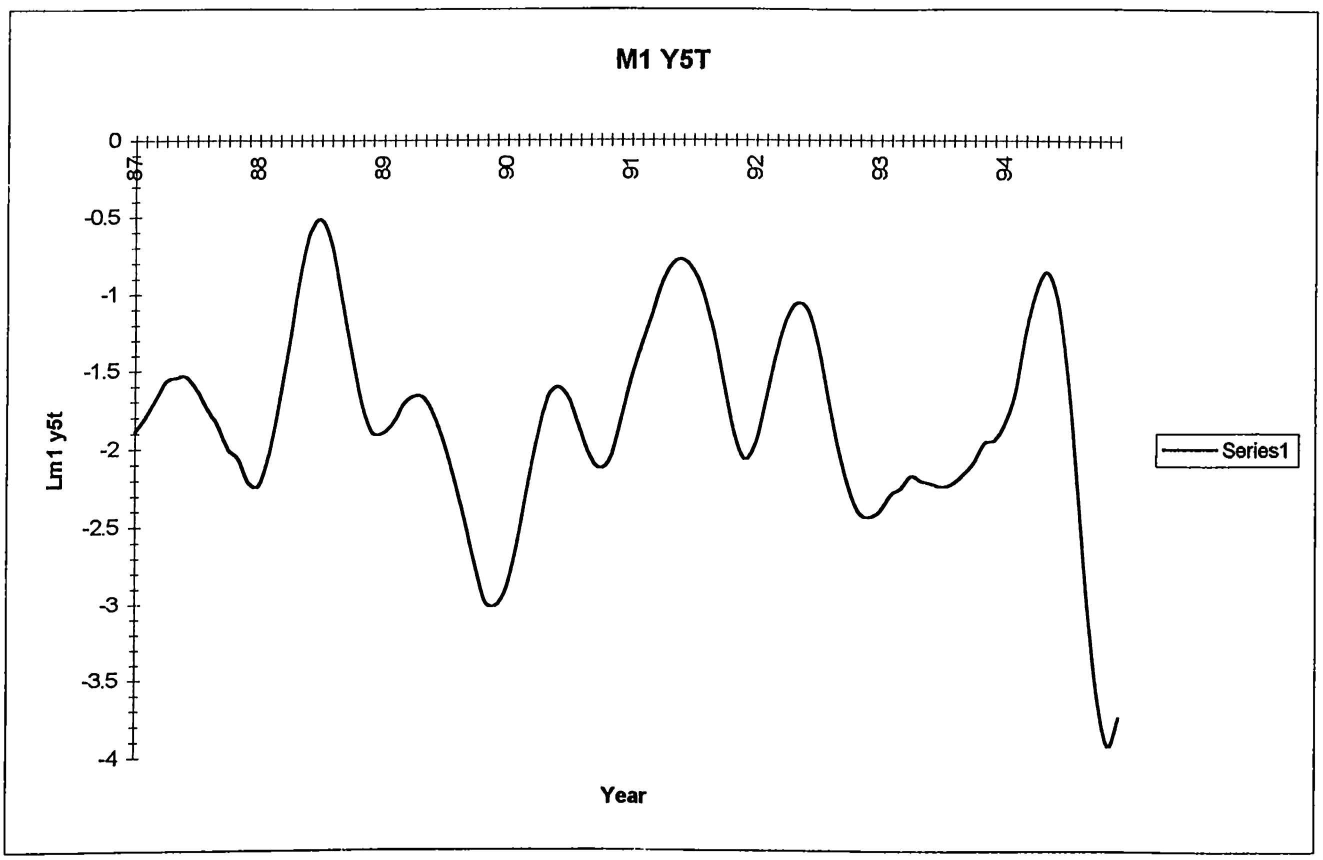
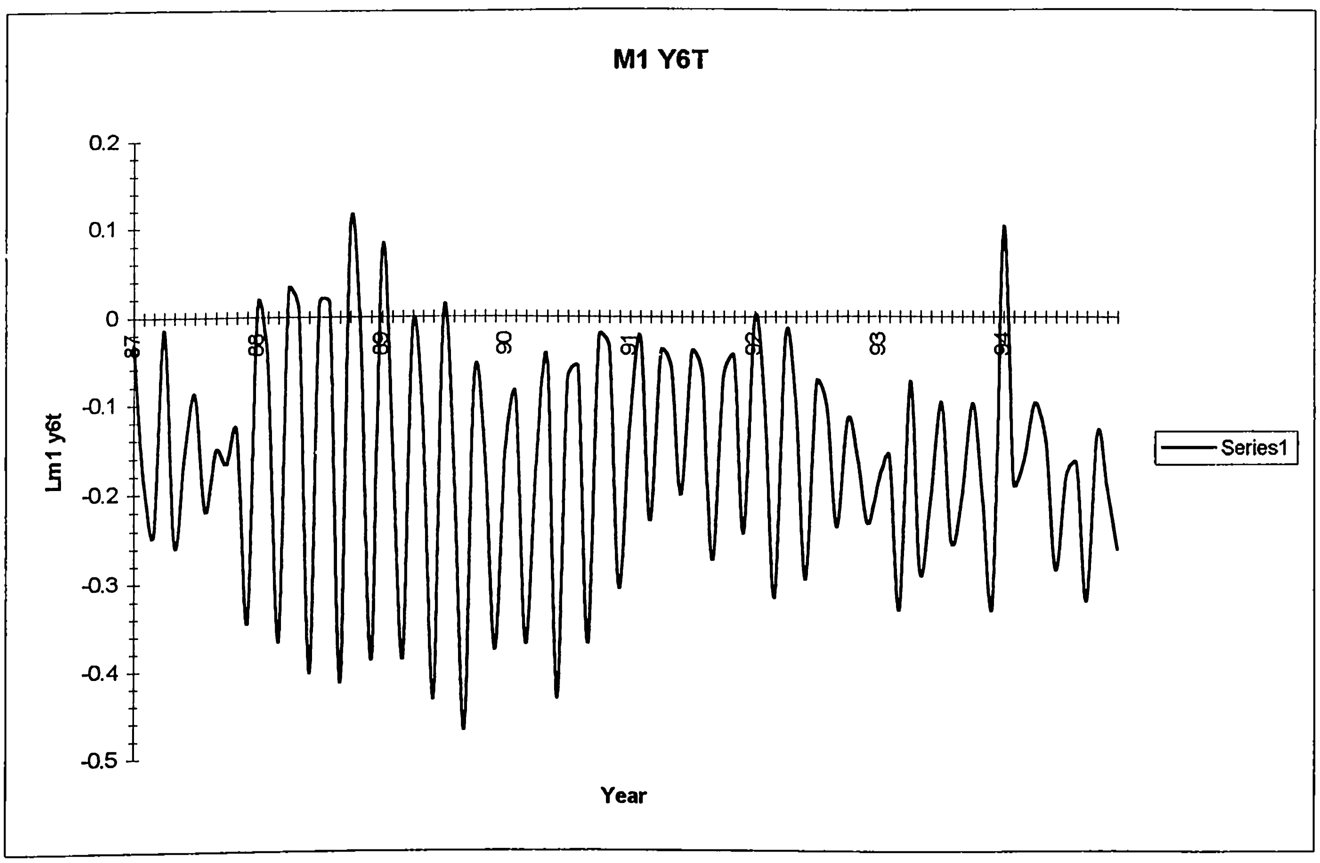


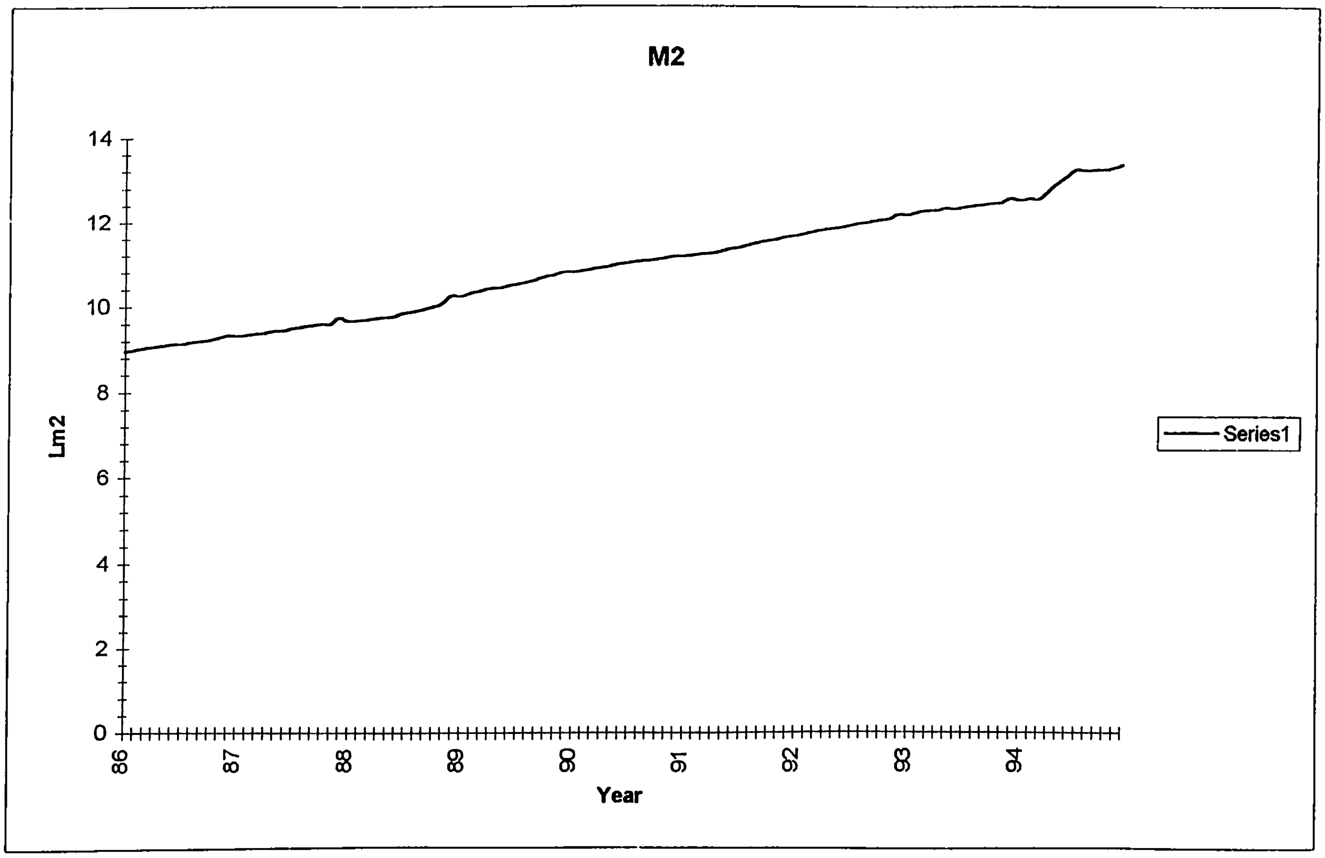
Table 15
M1 y5t for years between
87M1-94M12



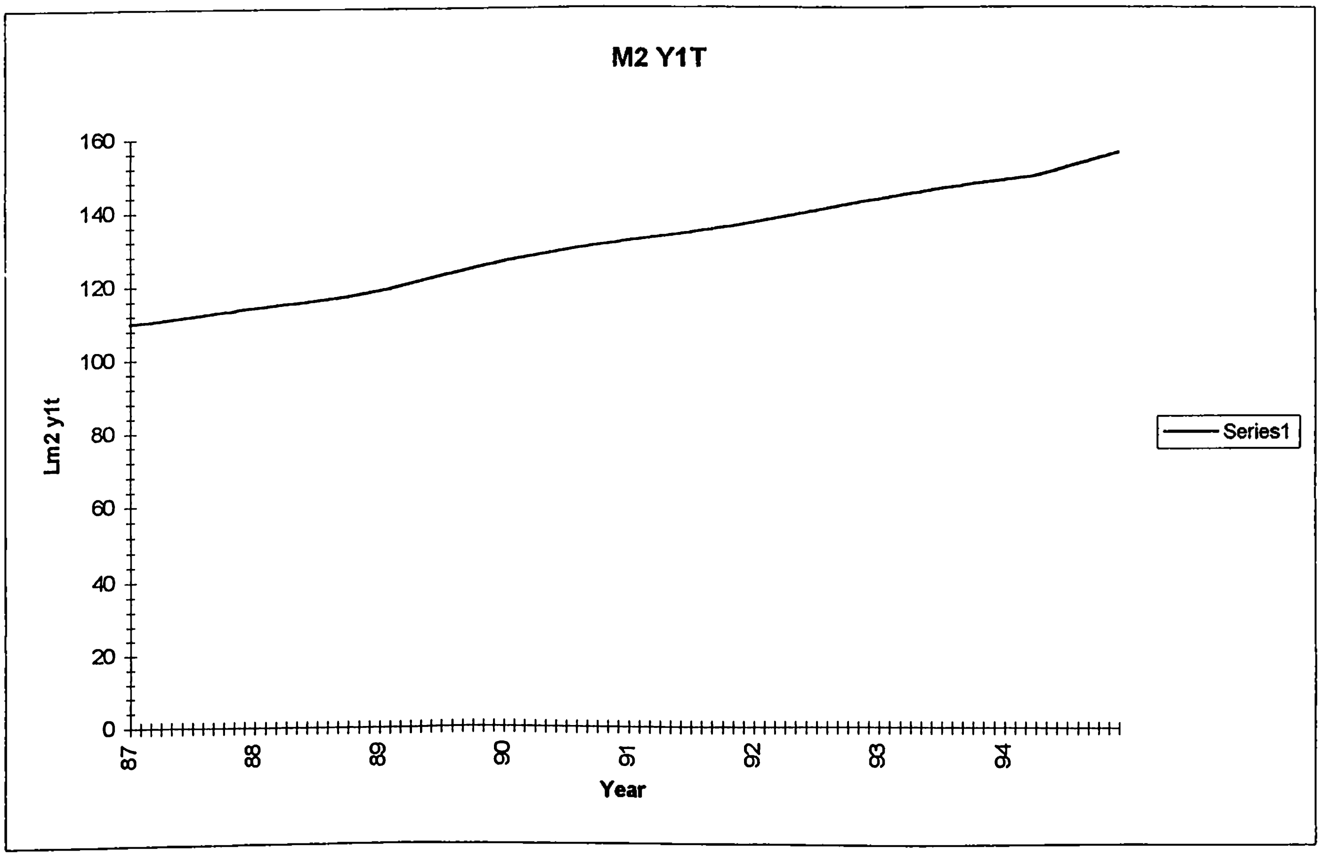
Graph 16
M1 y6t for years between
87M1-94M12



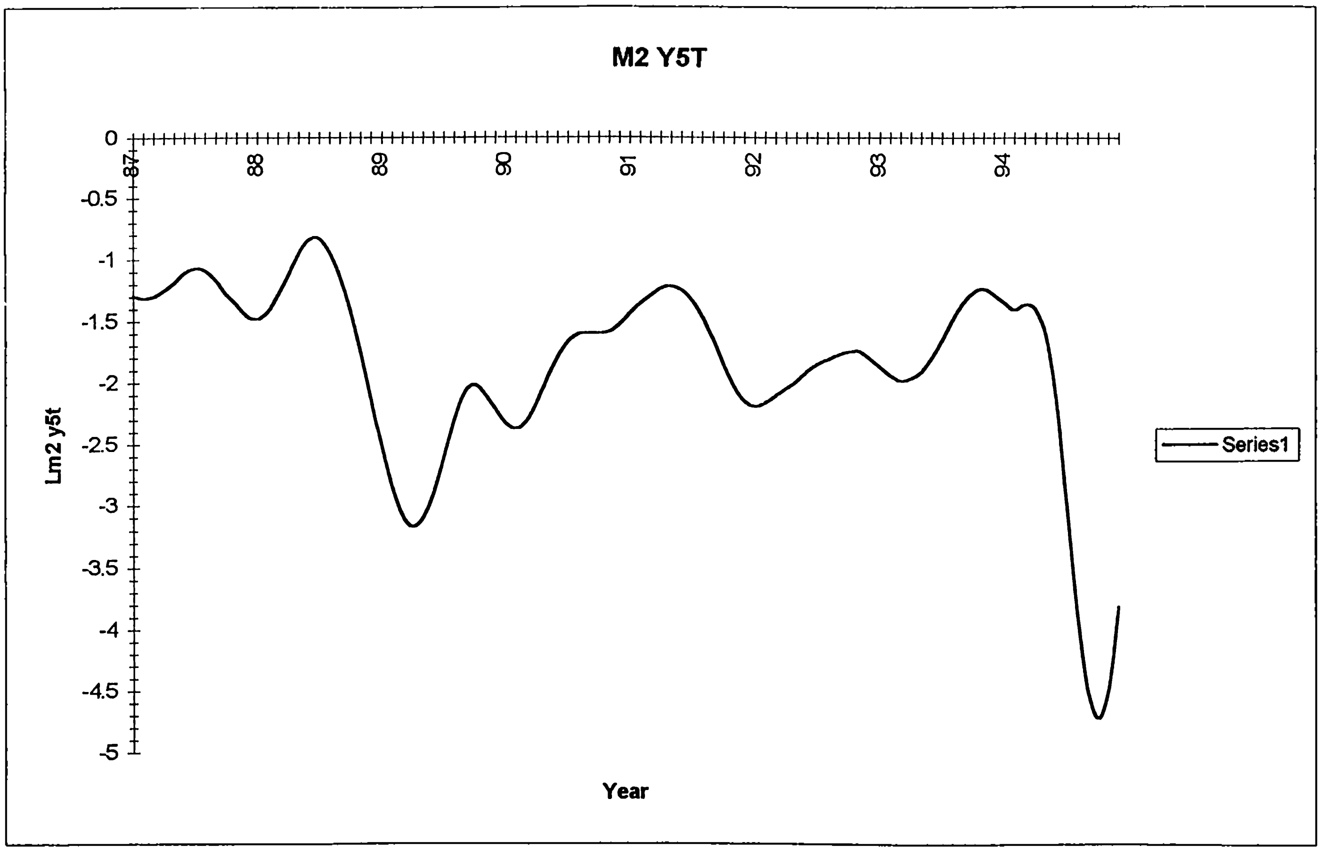
Graph 17
M2 for years between
87M1-94M12



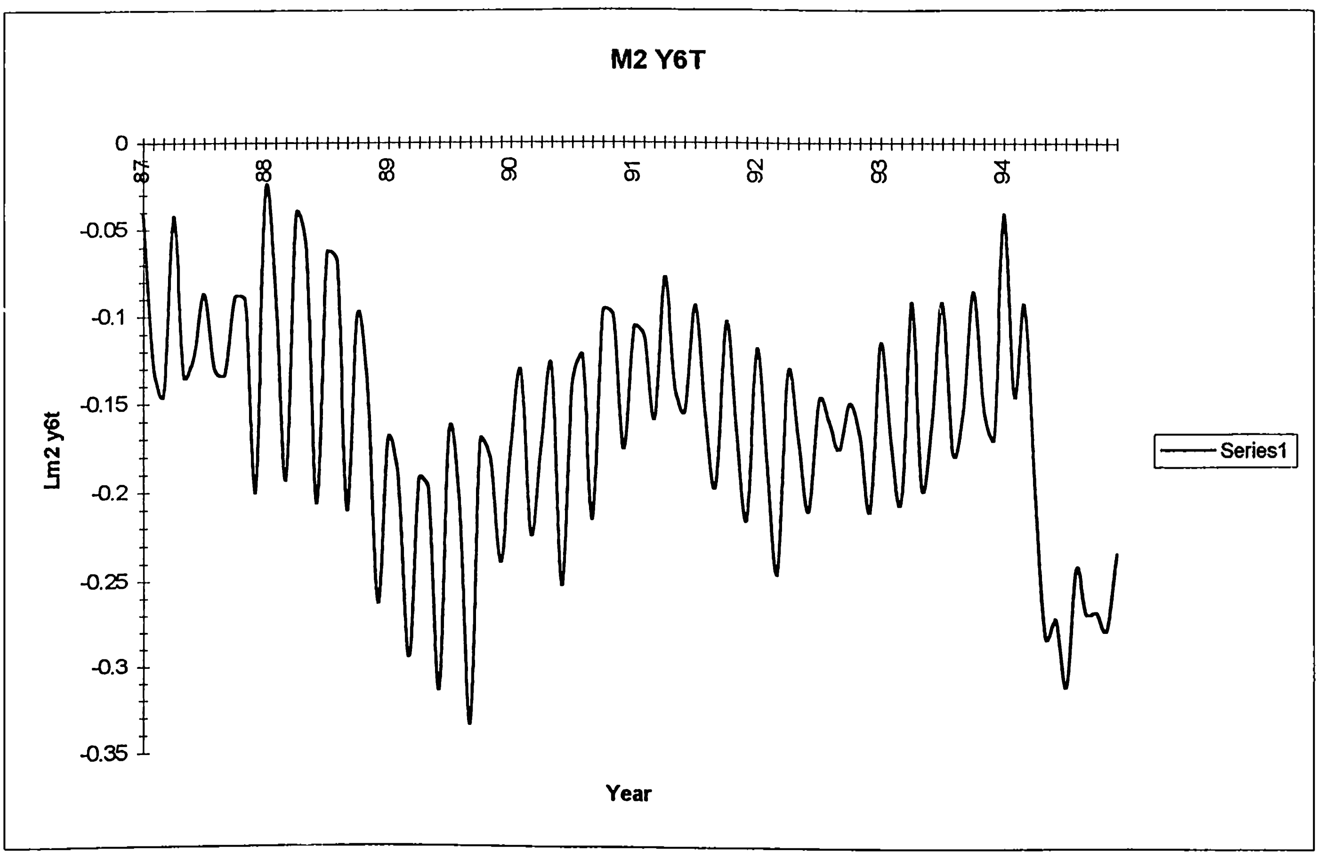
Graph 18
M2 y1t for years between
87M1-94M12



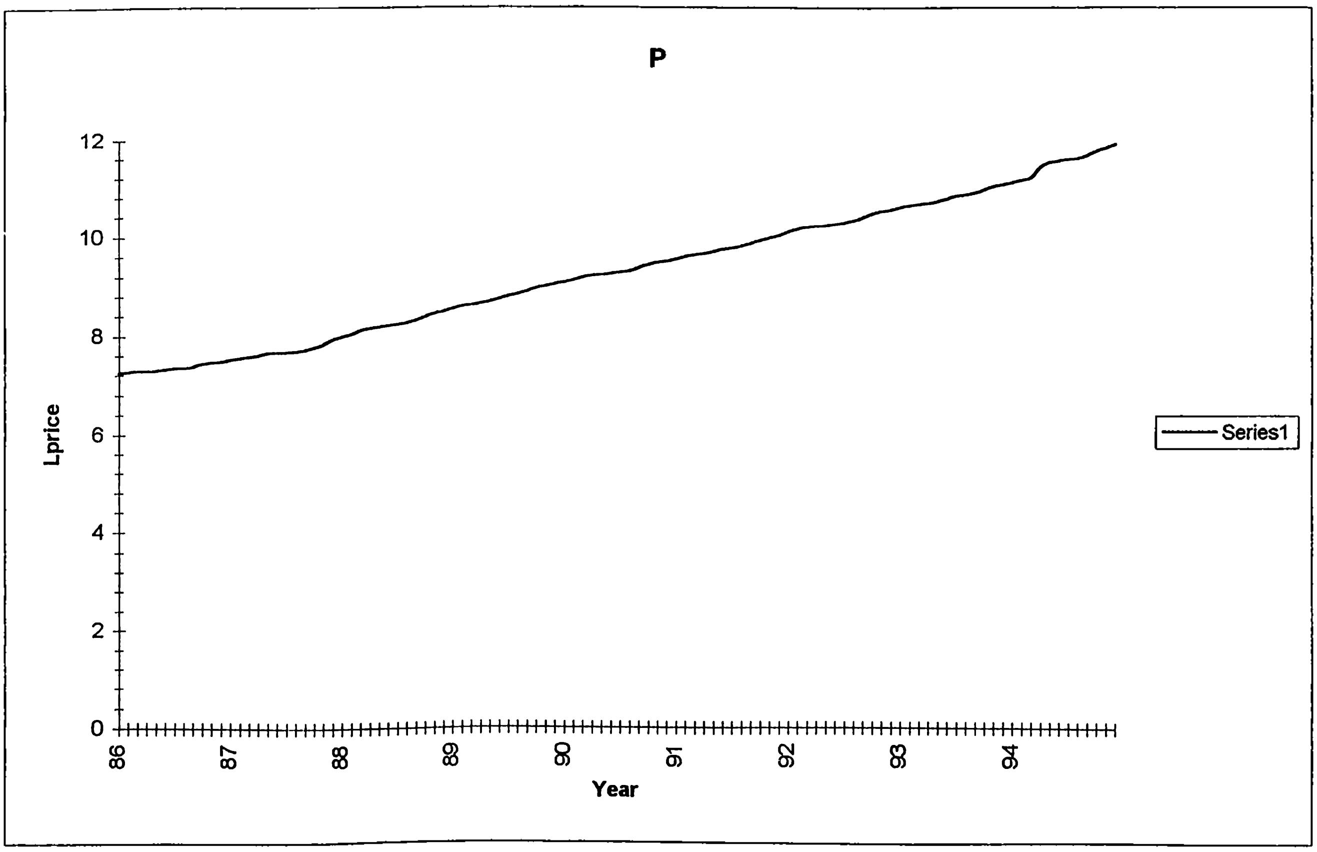
Graph 19
M2 y5t for years between
87M1-94M12



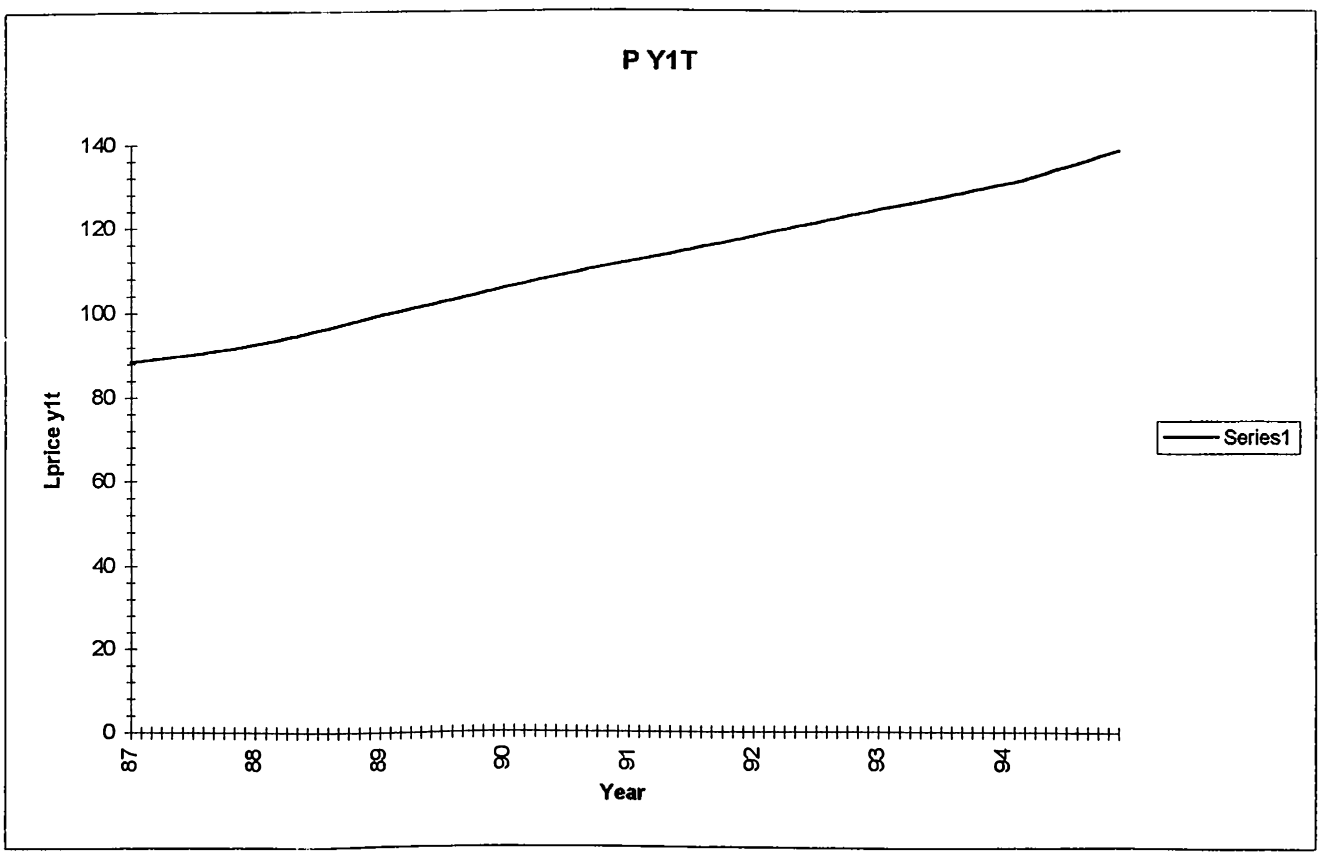
Graph 20
M2 y6t for years between
87M1-94M12



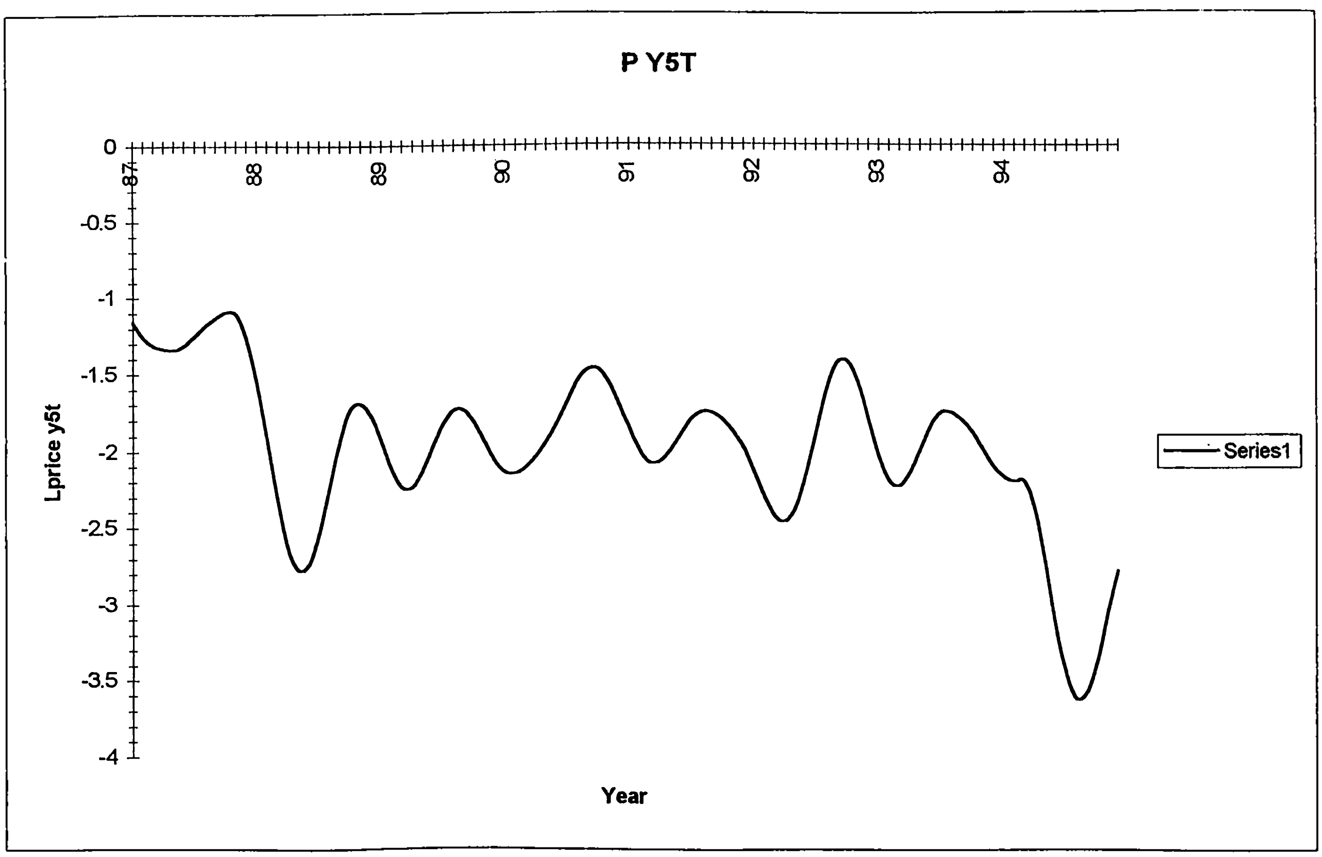
Graph 21
P for years between
87M1-94M12



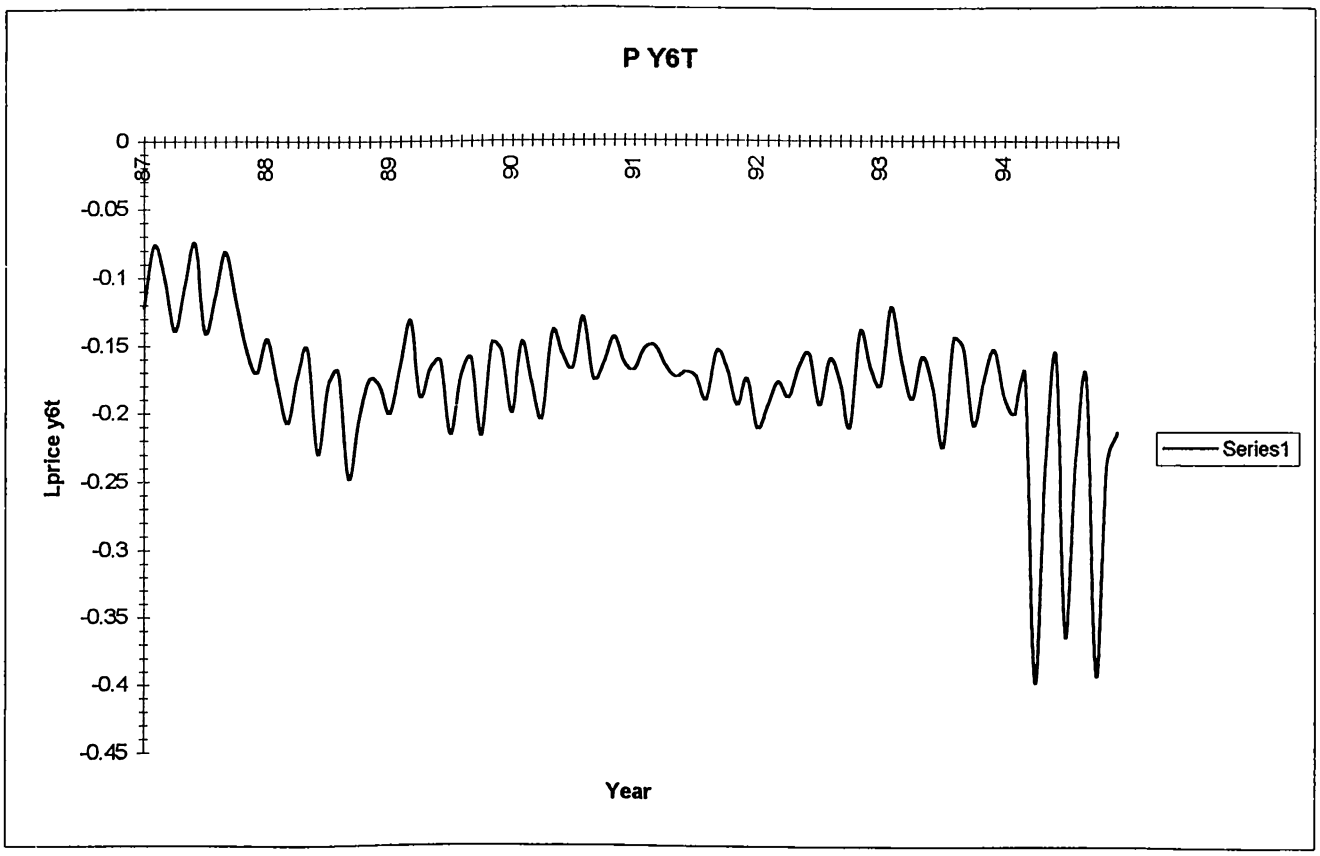
Graph 22
P y1t for years between
87M1-94M12



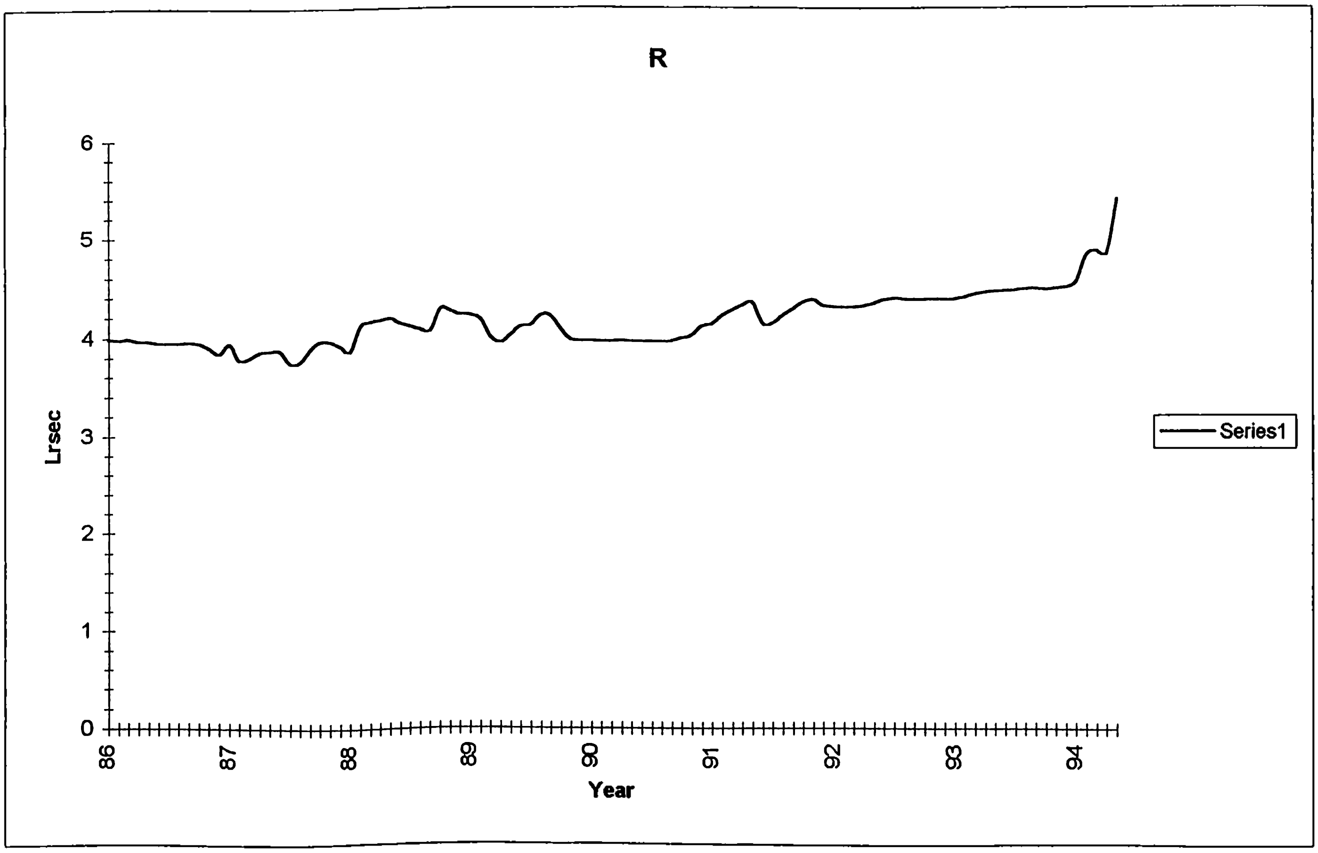
Graph 23
P y5t for years between
87M1-94M12



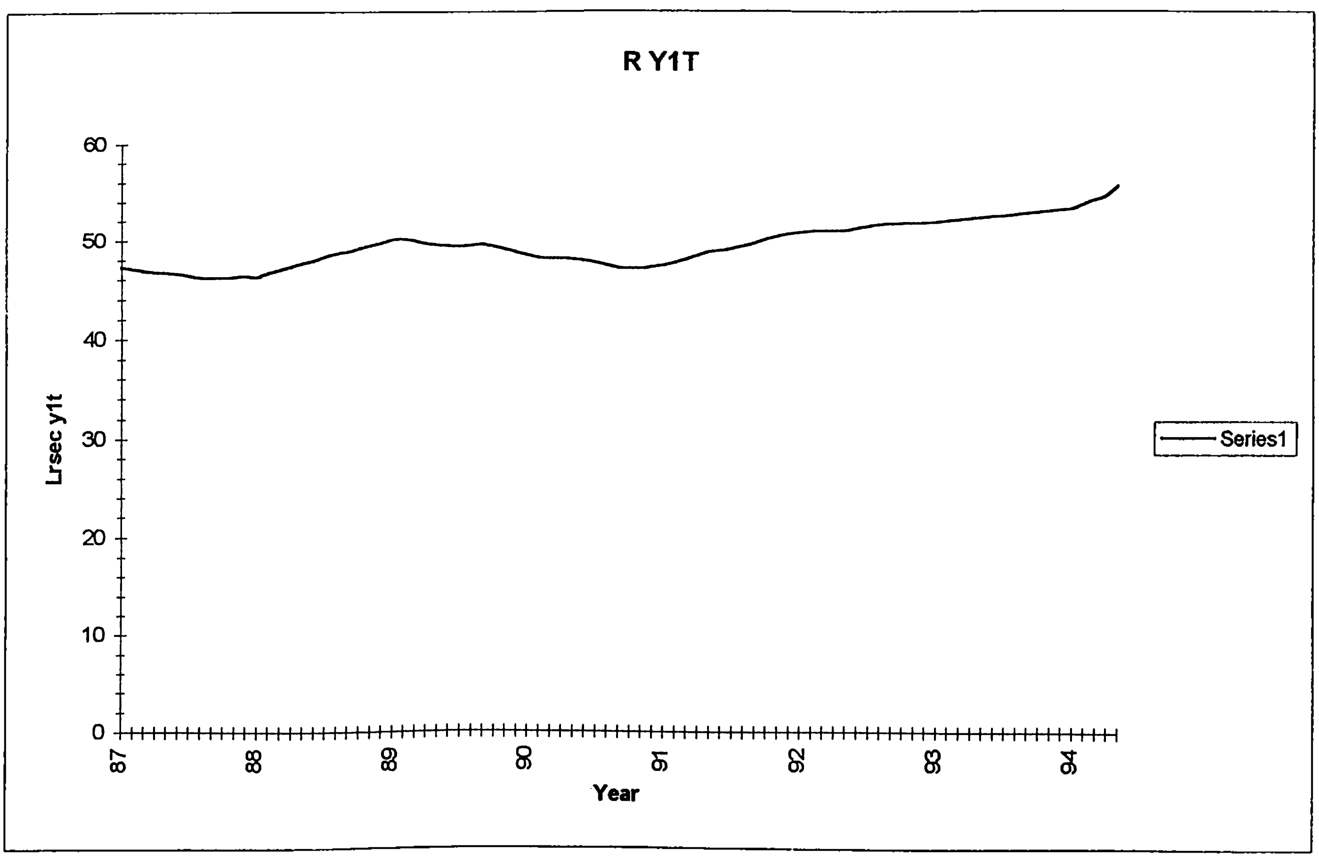
Graph 24
P y6t for years between
87M1-94M12



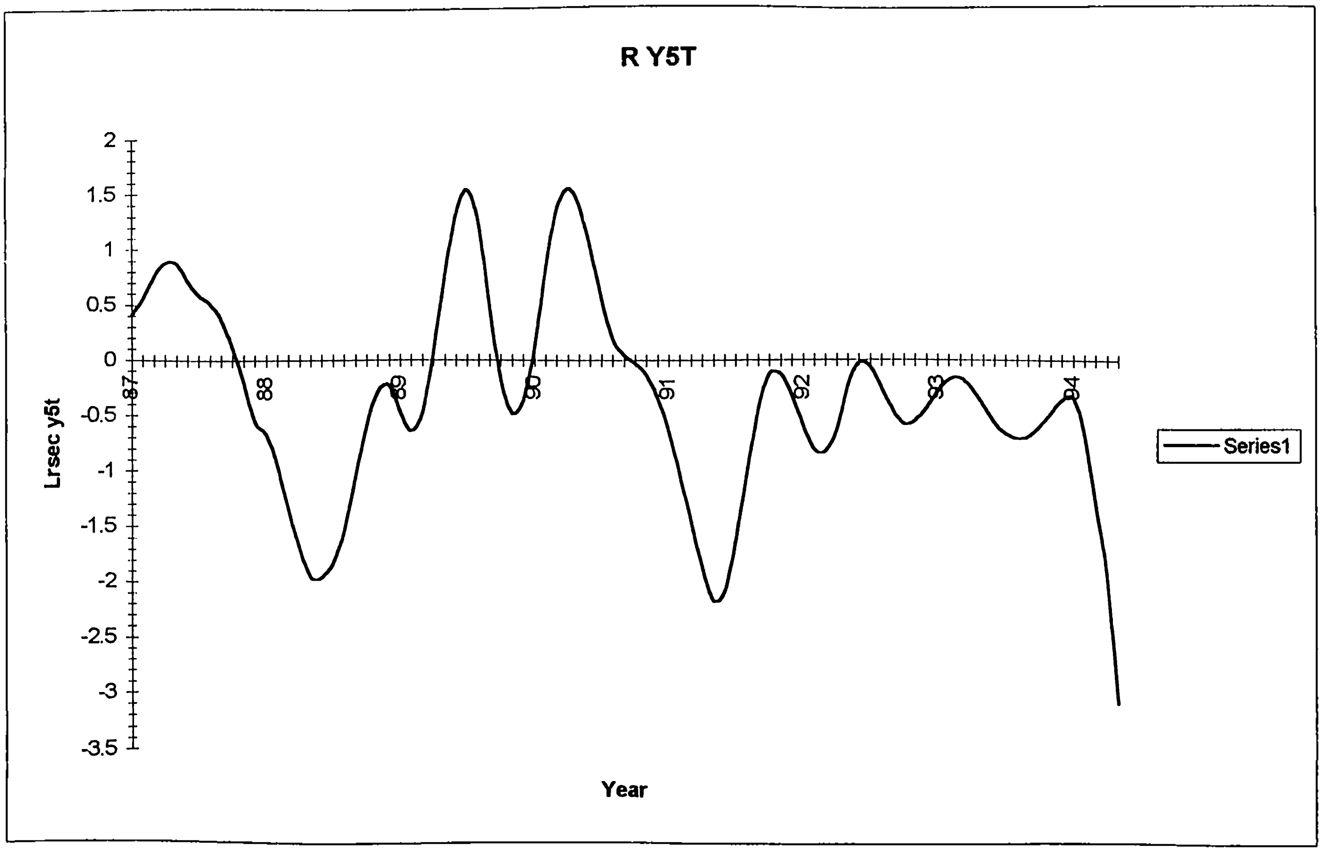
Graph 25
R for years between
87M1-94M5



Graph 26
R y1t for years between
87M1-94M5



Graph 27
R y5t for years between
87M1-94M5



Graph 28
R y6t for years between
87M1-94M5